Three principles of dynamics

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DIMES

Concept of Force

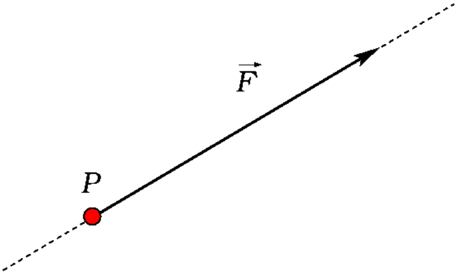
- Dynamics studies the effect of **forces** on the motion of bodies.
- It is based on **principles of dynamics**, codified by Newton towards the end of 1600
- Central concept is **force**, already known before Galileo
- Dynamics introduces new concepts: mass, momentum and strength

It was believed that the force was provoked by the motion, but Galileo overturned this paradigm introducing the idea that it is the force to cause a change in the state of motion.
The Aristotelian conception of motion assumed that in the absence of forces no motion should have been made, which is instead contradicted by the principle of inertia.

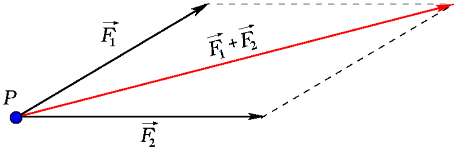
Definition

- We call **force** any cause that can change the speed of a body.
- Speed can vary both in module and in direction.
- A ball that bounces off the wall of a pool, changes the direction of its velocity and thus suffers a force.
- a body that performs a curved trajectory varies its velocity in the direction, not necessarily in the modulus
- The forces can also cause deformations.

Vector force

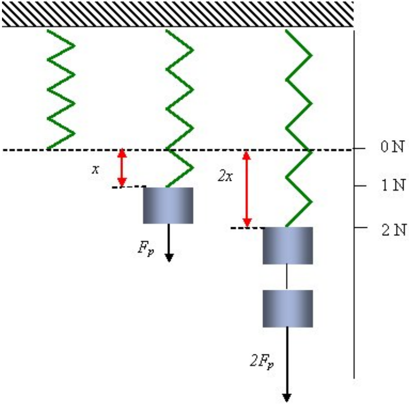


To completely specify a force, you must also know its point of application P.



Two forces $\vec{F}_1$ and $\vec{F}_2$ applied in the same point are equivalent to a single force obtained by adding $\vec{F}_1$ and $\vec{F}_2$.

How to measure forces



- dynamometer (elastic force)
- elongation is proportional to force
- $F \propto \Delta x$.
- $2F \propto 2\Delta x, 3F \propto 3\Delta x \cdots$.
- Weight: $\vec{F}_P = m\vec{g} = -k\Delta\vec{l}$

First principle

First principle (inertial principle)

- Reference systems **inertial** (which differ at most by a constant v) describe the same forces
- In the absence of forces, a body keeps its speed unchanged.
- Speed does not change direction and verse (so the motion is straight), nor in form (so the motion is uniform).
- In the absence of forces, a body moves with a straight and uniform motion.
- Special case: $\vec{v} = cost = 0$; in the absence of strength, a body at rest remains in stillness.

Inertial mass

In the interaction between two isolated material points, the ratio between the velocity variations is inversely proportional to the ratio of the respective masses.

$$\frac{\Delta v_1}{\Delta v_1} = -\frac{m_2}{m_1}$$

Inertial mass is a dynamic property (resistance to motion variation) while gravitational mass is a parameter associated with the gravitational pull force (the gravitational "charge" of a body). Numerically and dimensionally the two quantities coincide, unlike the electrostatic force in which the inertial mass does not coincide with the electrostatic charge that generates the force.

Second principle

Second principle

$$\vec{F} = m\vec{a}$$

If a force $\vec{F}$ acts on a body, the body gains an acceleration $\vec{a}$ proportional to $\vec{F}$:

$$\vec{a} = \frac{1}{m}\vec{F}$$

that is

$$\vec{F} = m\vec{a}$$

The constant m is called **inertial mass** of the body and is related to the its resistance to varying its speed (its own motion). SI unit is kilogram (kg)

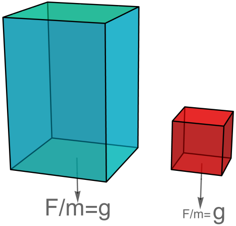
Newton

$$\vec{F} = m\vec{a} \Rightarrow 1N = 1kg \cdot \frac{1m}{1s^2}$$

- 1 Newton is the value of the force that gives a m 1 kg acceleration of 1 m/s^2.
- $\vec{F} = m\vec{a}$ is the **fundamental** equation of the dynamics: given the force, the acceleration is obtained and the equations of motion are obtained.
- $\vec{F}$ constant: material point moves uniformly accelerated (example $\vec{F_P} = m \cdot \vec{g}$) .
- If $\vec{F}$ is the force due to an elastic spring ($F_{EL} = -k \cdot \Delta x$), $\vec{a}$ is proportional to the spring's elongation, but directed in the opposite direction: the material point moves with harmonic motion.

Gravitational force: $\vec{F} = G\frac{m_1 m_2}{r^2} = m_1\vec{a} \Rightarrow \vec{a} = G\frac{m_2}{r^2}$

Ratio between weight and mass



If we consider different objects, with different masses, the ratio between force weight and mass remains constant, whatever the mass of the objects.

$$\vec{F} = G\frac{m_1 m_T}{r_T^2} = m_1\vec{g} \qquad \Rightarrow \qquad \vec{g} = G\frac{m_T}{r_T^2} \quad \Rightarrow \quad g = 9.81\frac{m}{s^2}$$

$(G = 6.67\times10^{-11}Nm^2kg^{-2}, r_T = 6353\,km, m_T = 5,9742\times10^{24}kg$
$)$

Frictions and constraints

kinetic friction force:

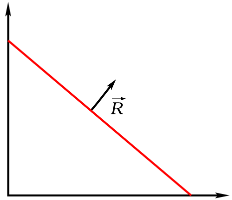
$$F_{ATT} = \mu_k F_\perp$$

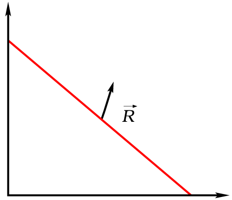
μ_k coefficient of friction and $F_\perp$ force perpendicular to the surface
fluid friction force:

$$F_{ATT} = -\beta \cdot v$$

where β is a coefficient that depends on the shape of the object and the nature of the medium in which the motion occurs, and v is the speed at which the object moves.

Smooth and not smooth constraint (with friction)





Frictions and constraints

If there is a friction force of the type $F_{ATT} = -\beta \cdot v$, the application of a constant force F_K does **not** produce a uniformly accelerated motion, but a uniform rectilinear motion, with a final speed equal to:

$$v_R = \frac{F_K}{\beta}$$

The final speed depends on the constant force impressed. If $F_K = mg$ the final speed of the body depends on its mass:

$$v_R = \frac{mg}{\beta}$$

Quantity of motion

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Second principle can be expressed in terms of quantity of motion

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{d\vec{m}}{dt}$$

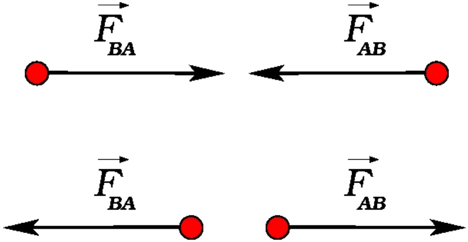
If m is constant: $\frac{d\vec{m}}{dt} = 0$ thus $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

Third principle

Third principle of dynamics

$$\vec{F}_{AB} + \vec{F}_{BA} = 0$$

Two point particles A and B:



If a body A exerts a force on a body B ,the body B exerts a force on A, equal in modulus and contrary in verse

Action and reaction

We note that, although equally intense, the two action-reaction forces can have very different effects:

$$\vec{F}_{AB} = m_B \vec{a}_B \qquad \vec{F}_{BA} = m_A \vec{a}_A$$

$\vec{a}_A$ and $\vec{a}_B$ are the accelerations of A and B produced by $\vec{F}_{BA}$ e $\vec{F}_{AB}$

$$\left|\vec{F}_{AB}\right| = m_B \left|\vec{a}_B\right| \quad \left|\vec{F}_{BA}\right| = m_A \left|\vec{a}_A\right|$$

thus

$$\frac{|\vec{a}_A|}{|\vec{a}_B|} = \frac{m_B}{m_A}$$

The accelerations caused in the two points are inversely proportional to their masses.

$\vec{Q} = \vec{q}_A + \vec{q}_B = cost$

- System of two masses m_A e m_B.
- **quantity of motion**:$\vec{q} = m\vec{v}$
- Total quantity of motion of the system $A + B$:
  $\vec{Q}_S = \vec{q}_A + \vec{q}_B = m_A\vec{v}_A + m_B\vec{v}_B$

The conservative statement (C) of the third principle states that the momentum of an **isolated system** (on which no external forces act) is constant.

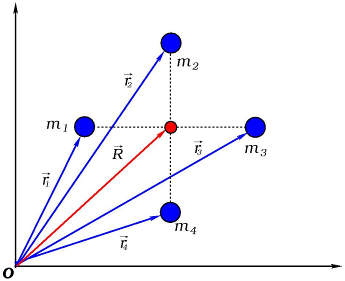
AR≡C

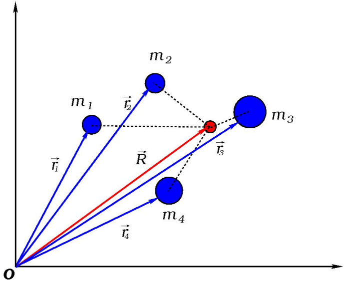
$$\vec{F}_{EXT} = 0 \Rightarrow \vec{Q}_S = cost$$

Center of mass

We define **center of mass or centroid** of a system of N material points the position:

$$\vec{R_{CM}} = \frac{m_1\vec{r_1}+m_2\vec{r_2}+\cdots+m_N\vec{r_N}}{M} = \sum_{i=1}^{N} \frac{m_i\vec{r_i}}{M} \qquad M = \sum_i m_i$$





It corresponds to the **average position** of all the material points of the system **weighted** on their mass.

Center of mass: speed

Analogously, for the speed of a system:

$$\vec{v}_{CM} = \sum_i \frac{m_i \vec{v}_i}{M}$$

- In an isolated system, the speed of the center of mass is constant
- the speed of the center of mass is given by the total momentum divided by the total mass:

$$v_{CM} = Q/M$$