Fluid Mechanics - Ideal Fluids

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DIMES

Fluids static

Ideal fluids

- Fluids are systems in liquid or gas phase
- No proper shape, but proper volume. They acquire the shape of their container. (they oppose no resistance to shear stress)
- Ideal fluid: incompressible with null viscosity. (interal friction)
- Real fluid (partly) compressible and viscous
- We will talk about fluid statics and dynamics
- Fluid characteristics are given by short-range intermolecular forces, that can transmit pressure in all directions.
- Gases have negligible interactions (are compressible), only have particle collisions.

intensive and extensive quantities

- with fluids, we don't have often clear borders to use when we want to describe them, everything can move
- we will work more with intensive quantities rather than extensive ones
- **extensive properties** is additive for subsystems (mass, volume, linear momentum, energy, etc. . .)
- **intensive properties:** does not depend on the size of the system (density, pressure, temperature)

density

- Fluid mass is distributed continuously (no point mass) so we introduce the *density*
- $\rho = \dfrac{m}{V}$
- $[\rho] = ML^{-3}$
- IS: measured in kg/m^3

Material	$\rho \quad (kg/m^3)$
Ether	736
Ethilic alcohol	791
Acetone	792
Benzene	809
Methilic alcool	810
Water	1000
Blood	1050
Mercury	13600

Table: Density of some fluids.

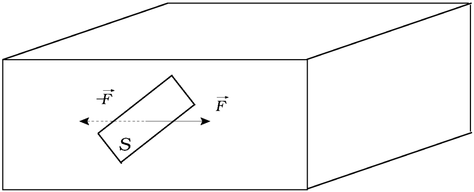
Pressure

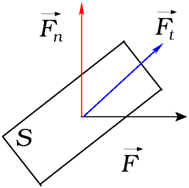
Pressure $P = \frac{F_n}{S}$

Pressure: ratio between force orthogonal to surface modulus $\vec{F}_n$ and surface A

$$P = \frac{\left|\vec{F}_n\right|}{A}$$

In IS measured in N/m^2: 1 N/m^2 = 1 Pascal (Pa).

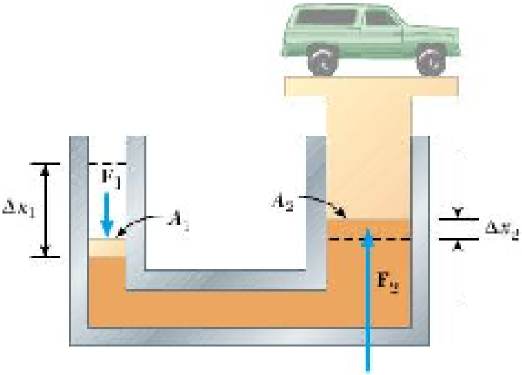




Pascal's principle

- when pressure is applied to a static fluid, it applies uniformly in every direction on every point of the fluid
- if there was a difference in pressure, there would be net force and movement of the liquid until the pressure balanced out
- this is how snorkling decompression works: you increase the pressure from the lungs (vertical) and it gets applied inside the ear canal (horizontal, in a different position)

Equilibrium between pressures

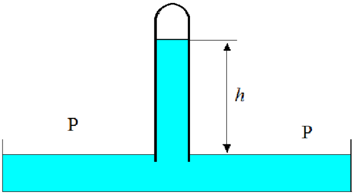


- $|\vec{P}_M| = 12.000N$, $A_2 = 0.1m^2$. Which force F_1 must be applied on surface A_1 of $0.002\ m^2$, to have equilibrium?
- $P_1 = P_2 \Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$
- $\Rightarrow F_1 = F_2 \frac{A_1}{A_2}$
- what happened to conservation of energy?
- $\Delta h_1 = \Delta h_2 \frac{A_2}{A_1}$
- the work is the same!

Pressure depends on fluid weight

- $F_P = m_L \cdot g \rightarrow P = \rho \cdot g \cdot h$
- m_L liquid mass, g gravity acceleration $m_L = \rho_L V_L$ thus weight can be expressed as $F_P = \rho_L V_L \cdot g$.
- Fluid volume is the product of surface S and depth h
- in a static fluid subject to gravity the pressure is equal in all the points of an horizontal plane
- in a static fluid subject to gravity the pressure, the pressure difference between two points at different heights is equal to the weight of a fluid column of unit section with height equal to the difference between heights

Torricelli's experience: $PA = m_{Hg}g$



- At equilibrium: $PA = m_{Hg}g$
- but $m_{Hg} = \rho_{Hg}V = \rho_{Hg}Ah$
- thus $P = \rho_{Hg}gh$
- P_{Atm}: 1 Atm=760 mm Hg

$$P = 13600\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 760\times 10^{-3}m \cong 133\times 760\ \frac{N}{m^2} = 101080\ Pa$$

Pressure measurement units

Torricelli's device was the first instrument to measure pressure.
NOTE: pressure measurement = height measurement
Other units:

- torr (o mmHg), pressure exerted by a 1 mm mercury column (133,3 Pa)
- baria, CGS (dina/cm^2), 1 baria=10^{-1} Pa
- bar, 1 bar = 10^5 Pa (millibar much used in meteorology).
- atm, $1atm = 1.013 \cdot 10^5 Pa$ (sea-level atmospheric pressure)
- mm H_2O, pressure exerted by a 1 mm water column (9,81 Pa)

Blood pressure - 1

- The maximum pressure at the peak of the sphyngotic wave is called systolic pressure, while the minimum blood pressure between two peaks is called diastolic pressure.
- For a young and healthy individual, systolic pressure is about $120 mmHg$ while the diastolic is about $80 mmHg$.
- The median pressure at the heart level is therefore 100 $mmHg$.
- If a person is in a stretched position, approximately, the average arterial pressure is equal to the cardiac pressure, ie 100 $mmHg$.

Blood pressure - 2

- In an upright position the pressure in different parts of the body is affected by the weight of the blood (hence its hydrostatic pressure): at the level of the head, 50 cm above the heart, the pressure will be:

$$P_{Head} = P_{Heart} - rho_{Blood}gh = 100mmHg - 39torr = 61mmHg$$

- On the other hand, if we measure the blood pressure at the feet, about 130 cm below the level of the heart, the pressure will be:

$$P_{Foot} = P_{Heart} + \ rho_{Blood}gh$$

$$= 100mmHg + 100mmHg = 200mmHg$$

Ideal fluid motion

flow rate

To describe the motion of a fluid, we consider the displacement of its volume in the time unit:

$$Q = \frac{\Delta V}{\Delta t}$$

The volume of a fluid inside a conduit is given by the product for the S section of the conduit for the Δ distance traveled therein:

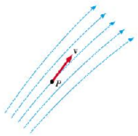
$$V = S \cdot \Delta l$$

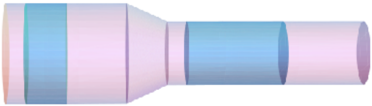
thus

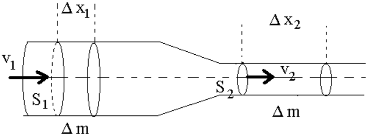
$$Q = \frac{\Delta V}{\Delta t} = S\frac{\Delta l}{\Delta t} = S \cdot v$$

NOTE: for an ideal fluid the velocity is the same in all the points of a section of the conduit

Conservation of flow rate







- An ideal fluid is incompressible: mass (and volume) are preserved during the motion
- $\Delta x = v \cdot \Delta t$
- $V_1 = S_1 \cdot v_1 \Delta t; \quad V_2 = S_2 \cdot v_2 \Delta t$
- $m_1 = \rho S_1 \cdot v_1 \Delta t; m_2 = \rho S_2 \cdot v_2 \Delta t$
- $V_1 = V_2 \rightarrow S_1 \cdot v_1 = S_2 \cdot v_2$
- In the smaller tube sections the speed increases (neglecting friction)

work of a fluid

$$\Delta L = F\Delta l = P \cdot S \cdot \Delta l = PV$$

it's better expressed with pressures and volumes, rather than on forces and displacements

power of a pump

$$W = \frac{\Delta L}{\Delta t} = P\frac{\Delta V}{\Delta t} = P \cdot Q$$

Thus the power depends on the pressure on the fluid along the conduit and on its flow (amount of fluid displaced in the time unit).

Heart as a pump

- Under normal conditions the heart pumps around 6 liters per minute ($10^{-4}m^3/s$)
- the average pressure of 100 mmhg is equal to $1.3 \cdot 10^4$ Pa
- this means that the average power is roughly $1.3W$

Bernoulli

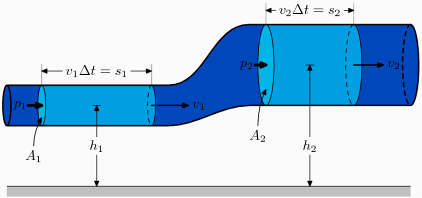
Bernoulli's theorem

In an ideal fluid with no friction only conservative forces act $\rightarrow$ total mechanical energy is preserved

Energy depends on

- fluid velocity (E_C)
- fluid weight (U_G)
- works applied to the fluid (e.g. pumps: ΔL)

Bernoulli's Theorem



- $L = p_1V_1 - p_2V_2 + mgh_1 - mgh_2$
- $L = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Conservation of total mechanical energy

$$p_1V_1 + mgh_1 + \frac{1}{2}mv_1^2 = p_2V_2 + mgh_2 + \frac{1}{2}mv_2^2$$

- dividing by V : $p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$
- equals to: $P + \rho g h + \frac{1}{2}\rho v^2 = \text{cost}$

Bernoulli's theorem: $P + \rho g h + \frac{\rho v^2}{2}$=cost

Bernoulli's theorem is a direct application of the principle of conservation of mechanical energy

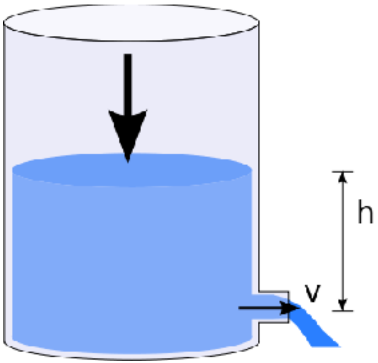
- flow rate is preserved
- energy is preserved
- speed depends only on tube section
- pressure P varies under several conditions (height, tube size, etc...)

virtual pipes

- Bernoulli theorem does not apply only in real pipes, but also in general free fluid motion
- sea and air currents respect it as well
- each flow stream could be treated as its own pipe

Applications

Torricelli's theorem

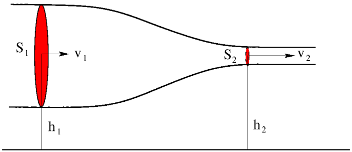


- upper section: v=0 and $U = mgh$
- lower section: v=max and $U = 0$
- $\rho gh = \frac{\rho v^2}{2} \rightarrow v = \sqrt{2gh}$
- the expression of the speed is the same as for a free falling object!

Venturi effect

If I reduce the section of the conduit, the pressure of the liquid increases:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$



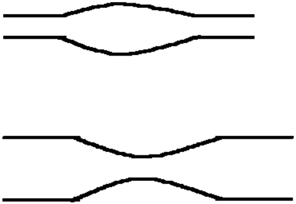
$$h_1 = h_2 \Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

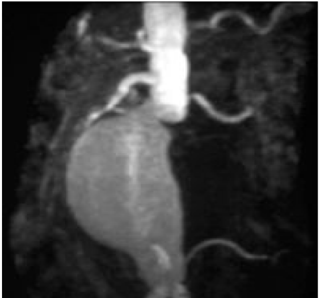
$$S_1 v_1 = S_2 v_2 \Rightarrow v_2 = \frac{S_1}{S_2} v_1$$

$$\Delta P = \frac{1}{2}\rho v_1^2 (\frac{S_1^2}{S_2^2} - 1) > 0$$

$$P_1 > P_2$$

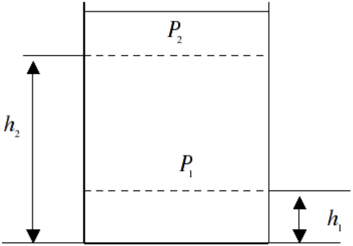
Venturi effect in real life





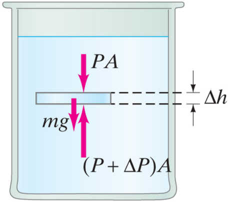
Archimede

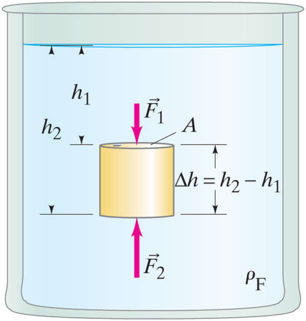
Stevino's law: $\Delta P = \rho g h$



- stationary conditions $(v_1 = v_2 = 0)$
- $P_1 + \rho g h_1 = P_2 + \rho g h_2$
- $P_1 - P_2 = \rho g \left(h_2 - h_1\right)$
- $\Delta P = \rho g \Delta h$

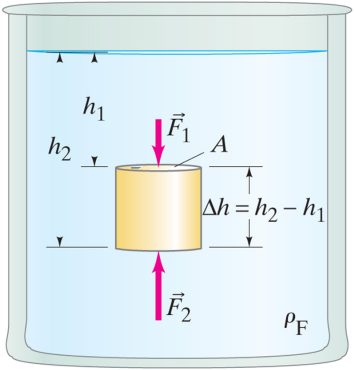
Archimede: $F_A = \rho_L V g$





- Fluid foil in equilibrium (solidification principle)
- Pressure in a fluid increases with depth
- The pressure is perpendicular to the surface (Pascal)
- On the upper surface $P_1 = \rho g h_1$ down
- On the lower surface $P_2 = \rho g h_2$ upwards
- then $\Delta P = \rho g(h_2 - h_1) = \rho A \Delta h = m_f g$
- If, instead of a fluid at rest, we have a solid with ρ_s $(\rho_S > \rho_L, \rho_S = \rho_L \rho_S < \rho_L)$

Archimede's principle



- Weight $F_P = mg = \rho_C V g$
- Pressure $\Delta P = \rho_L g \Delta h$
- Archimede force $F_A = P \cdot S = \rho_L V g$
- result $\Delta F = (\rho_C - \rho_L) V g$

A body floats or sinks according to its **density**

Archimede's principle

If a body floats, will emerge up to the height in which Archimede's force and weight of the submerged side coincide.

$$\Delta F = 0$$

$$m_C g = V_C \rho_C g = V_{IMM} \rho_L g$$

$$V_C \rho_C = V_{IMM} \rho_L$$

$$\frac{V_{IMM}}{V_C} = \frac{\rho_C}{\rho_L}$$

Archimede's principle

- If an object is immersed in a fluid its weight seems lower than outside the fluid.
- Floating derives from the balance of two forces: the weight force, which pushes the object downwards and a floating force, exerted by the liquid, which pushes the object upwards.
- the pressure in a fluid increases with depth (increasing the mass of overlying liquid) so that the pressure on the lower face (directed upwards) of the cylinder is greater than that on the upper face (directed downwards).

exercises

pressure at the bottom of a pool

- A pool of size 24m x 24 m and with a depth of 2.4 meters is filled completely with water
- what is the pressure at the bottom of the pool?
- what would it be if it was filled with mercury? $(1.36 \cdot 10^4 kg/m^3)$

pressure at the bottom of a pool - solution

- the pressure at the bottom is the sum of the atmospheric pressure and the increase of pressure due to the water column
- the water column depends only on the depth of the pool:

$$P_{bottom} = P_{surface} + \rho_w g h$$

$$P_{bottom} = 10^5 + 10^3 \cdot 9.8 \cdot 2.4 \sim 1.24 \cdot 10^5$$

- in the case of mercury one replace only the density in the expression:

$$P_{bottom} = 10^5 + 1.36 \cdot 10^4 \cdot 9.8 \cdot 2.4 \sim 4.2 \cdot 10^5$$