Fluid Mechanics - Viscous Fluids

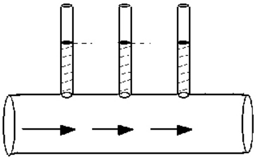
Enrico Giampieri

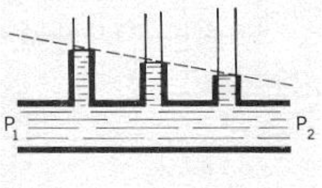
DIMES

Viscosity

Real fluids: viscosity

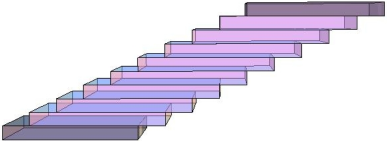
- In a real fluid the internal frictions (between fluid sheets) are not negligible $\Rightarrow$ the motion of the fluid changes.
- To keep a fluid moving in a horizontal conduit, a pressure difference ΔP is needed at its ends
- The **hydrodynamic resistance** typical of each fluid can be defined
- The phenomenon of viscosity is complex, not yet fully understood





Frictional force $F = \eta \cdot A\frac{\Delta v}{\Delta z}$

Force exerted between two foils

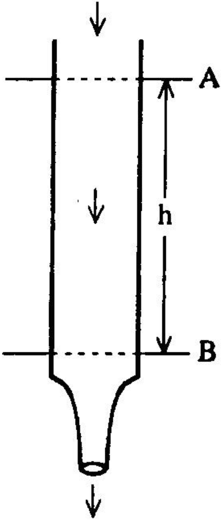


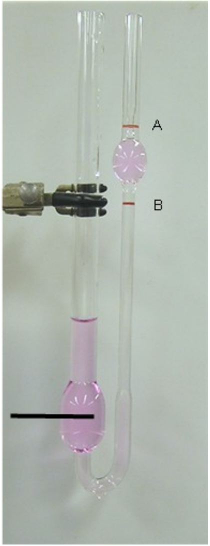
- depends on speed difference Δv
- depends on foil distance Δz
- is proportional to contact surface A
- speed decreases with Δz.

- Friction forces depend on microscopic energy exchanges between fluid particles in different foils (at different speed) because of the collisions between them.
- The viscosity η is measured in $Pa \cdot s$ (Poise).

Viscosimeters

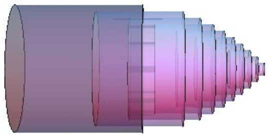
Measure the speed of the fluid as a function of pressure: $v \sim \eta^{-1}$

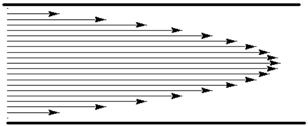




Laminar motion

Laminar motion in real fluids





- In a real fluid the velocity is not the same in all the points of a section
- Coaxial cylindrical layers of radius r between 0 and R.
- Speeds vary according to the distance from the axis.
- v maximum on the $r = 0$ axis
- $v = 0$ on the wall $(r = R)$

$$v(r) = \frac{1}{4\pi} \frac{\Delta P}{l} \left(R^2 - r^2\right)$$

Poiseuille's law: $\Delta P = R \cdot Q$

- in a duct where a viscous fluid flows under a laminar regime, fixed the other parameters, the flow rate increases with the fourth power of the section beam.

$$Q = \frac{\pi}{8} \frac{\Delta P}{l} \frac{r^4}{\eta}$$

- ΔP is the pressure variation (ie loss of load).
- l is the length of the conduit.
- η is the dynamic viscosity of the fluid considered.
- By defining $R = \frac{8l\eta}{\pi r^4}$ we get $\Delta P = R \cdot Q$ which is formally analogous to Ohm's law.

Poiseuille and blood flow

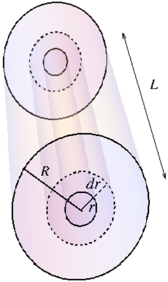
- If the radius of the arteries is reduced (cholesterol deposition), the hydrodynamic resistance increases, and therefore, to maintain the same flow rate the pressure must be increased.
- *E.g. if the radius decreases by* 10%*, the heart must increase the pressure by a factor of* 1.5 *to maintain the same range:*

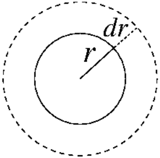
$$\frac{1}{0.9^4} = 1.52$$

- an increase in blood pressure (to keep the flow constant, and therefore the flow) involves more work of the heart

$$\Delta W = \Delta P \cdot Q$$

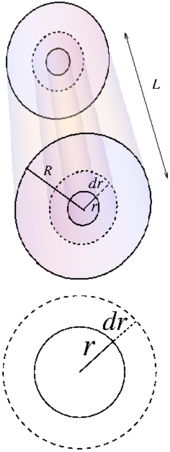
Proof of the Poiseuille law - 1





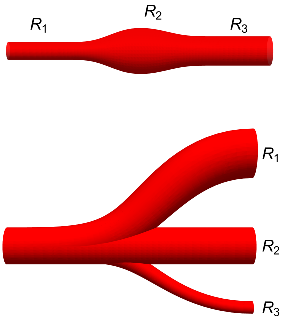
- inside cylinder, radius r, length L
- on it act two forces:
- driving force due to pressure difference $F_P = \Delta P \pi r^2$
- friction force F_A applied on the surface of the cylinder $F_A = -\eta 2\pi r L \frac{dv}{dr}$
- in stable conditions $F_A + F_P = 0 \Rightarrow dv = \frac{\Delta P r}{2\eta L} dr$.
- solving the integral: $v = \frac{\Delta P r^2}{4\eta L} + K$, with K an arbitrary constant.

Proof of the Poiseuille law - 2



- the constant K can be determined setting the speed of the outer layer of fluid equal to 0, obtaining $v(r) = \frac{\Delta P}{4\eta L}(R^2 - r^2)$.
- remembering that flow rate is $Q = vS$ and thus $dQ = v\,dS$, we can integrate the speed over all internal cylinders
- $\frac{\Delta P}{4\eta L}(R^2 - r^2) \cdot 2\pi r \cdot dr$ between 0 and R
- we obtain $Q = \frac{\Delta P \pi R^4}{8\eta L}$
- if we define $\frac{\pi R^4}{8\eta L} = 1/R_{IDR}$
- we obtain $Q = \frac{\Delta P}{R_{IDR}}$

Parallel and serial hydrodynamic resistances



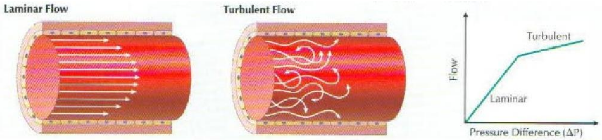
- the total resistance of n pipes in series is given by the sum of their resistances $R_{\text{eq}} = R_1 + R_2 + \cdots + R_n$
- the total resistance of n pipes in parallel is given by the inverse of the sum of inverse of their resistances $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$
- for only two pipes we obtain $R_{\text{eq}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$
- if they are identical: $R_{\text{eq}} = \frac{R}{2}$
- resistance increase with serial pipes, and decreases with parallel ones

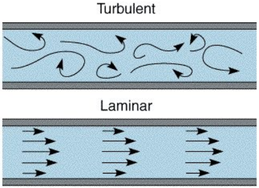
Turbulent motion

Turbulent motion

- The above considerations are valid under laminar flow conditions. But the motion of real fluids can be turbulent.
- The turbulence appears when the fluid has a high velocity compared to its viscosity
- in this case the current lines acquire a complex structure and the Poiseuille law is no longer valid.
- As the speed increases, vortexes begin to form. Since the velocity in the vortexes is high, there is correspondingly a decrease in pressure (pressure resistance).
- Moreover, the existence of lines of velocity along directions different from that of the conduit show how it becomes increasingly difficult for the fluid to proceed along the conduit (at the same pressure applied).
- The conditions in which the whirling motion begins to occur depend on the velocity of the fluid, the density, the viscosity, and the shape of the conduit.

Turbulent motion: Reynolds' number





- Reynolds' number: $R = \frac{\rho v D}{\eta}$
- v average fluid speed
- η viscosity, ρ fluid density
- D tube diameter
- $R < 2000$ laminar motion
- $R > 2000$ turbulent motion
- in between there are fluctuating behaviors
- measures the ratio between the inertia of the fluid and the viscosity

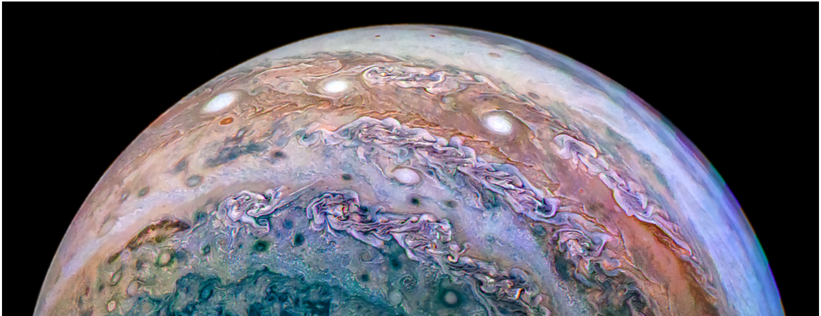
Turbulence in blood flow

Vessel	speed (cm / sec)	Diameter (cm)	Reynolds Num.
Aorta	48	2.5	3400*
Artery	45	0.4	500
Arteriole	5	0.005	0.7
Capillary	0.1	0.0008	0.002
Venule	0.2	0.002	0.01
Vein	10	0.5	140
Vena Cava	38	3.0	3300*

taken from "Hemodynamics", by Dr. D. Penney

Eddies

- Turbulence is characterized by the presence of "eddies", vortexes of fluid of various sizes
- each eddie break down and passes its energy to smaller and smaller ones, until all the energy is dispersed as thermal energy
- this is defined turbulent cascade, and is present across several order of magnitude



Sphygmomanometer

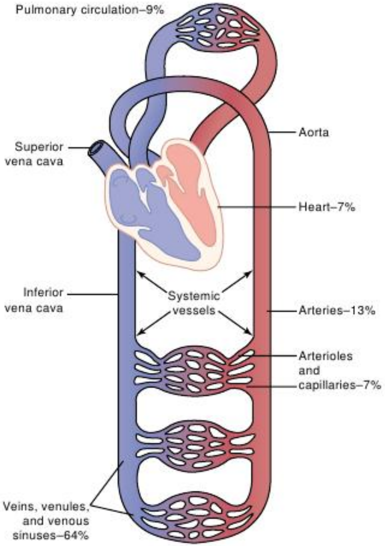
- turbulence is what allows to measure the pressure and hearth beat of a patient with a stethoscope and a Sphygmomanometer
- when the pressure raises above the systolic values, the artery is closed and no blood passes.
- when the pressure is below the diastolic values, the artery is always open and the blood flows more or less normally
- the the pressure is between the systolic and the diastolic value, the blood passes in short bursts. the speed generates turbulence and creates a noise that can be heard, one for each heart beat.
- by starting from a high pressure and reducing it, we can measure the systolic and diastolic pressure by checking at which pressure the sounds started and stopped

vases shear stress

- when a stenosis is present, there might be turbulence after it, due to the speed increase
- this leads to a strong stress on the tissues, and can lead to even more damages after the stenosis

blood circulation

Distribution of blood volume



Vessel	Cross-Sectional Area (cm^2)
Aorta	2.5
Small arteries	20
Arterioles	40
Capillaries	2500
Venules	250
Small veins	80
Venae cavae	8

- Vein area is $\approx$ 4 times arteries ($\Rightarrow$ contain more blood)
- How do we calculate speed?

Blood Flow Rate

- constant flow rate $\Rightarrow v \approx constant/A$ (inverse proportionality).
- at rest $v_{AORTA} \approx 33cm/sec$, $v_{CAPILLARIES} \approx v_{AORTA}/1000, \approx 0.3mm/sec$.
- since $l_{CAPILLARIES} \approx 0.3 - 1mm$, the blood remains in the capillaries for 1-3 sec.
- very short time for exchanges

Pressure Distribution

- The heart pumps blood continuously and then, on average $P_{AORTA} \approx 100mmHg$
- Because of the pulsatility, $P_{DIAST} = 80mmHg < P_{ARTERIOSA} < P_{SIST} = 120mmHg$
- At the end of the vena cava the pressure falls to 0 mm Hg
- $P_{VENUES} \approx 10 - 17mmHg <= P_{CAPILLARIES} <= P_{ARTERIOLE} \approx 35mmHg$
- P_{VENULE} is low enough to let the plasma out of the capillaries and thus also exchange nutrients.
- Pulmonary arterial pressure is pulsatile, as in the aorta, but less: the systolic pressure of the pulmonary artery is 25 mm Hg, while the diastolic pressure is 8 mm Hg.
- Capillary pulmonary pressure is, on average, only 7 mm Hg, sufficient to expose the blood in the lungs to oxygen.

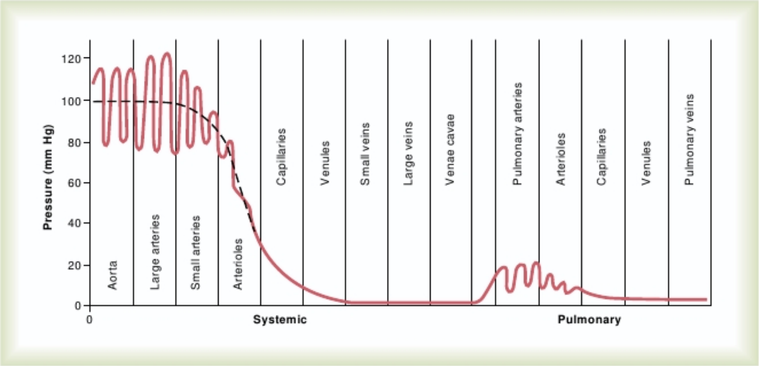


Figure 14–2

Normal blood pressures in the different portions of the circulatory system when a person is lying in the horizontal position.

Blood flow rate management

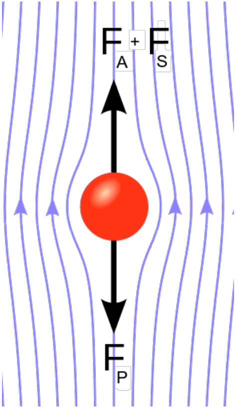
- The rate of blood flow in each tissue depends on its state: if it is active it recalls a flow 20-30 times greater than the basal state, but the heart can increase its output only 7 times.
- Therefore it is not possible to increase only the cardiac output and it becomes necessary to control the local microvasal dilatation both by nutrients sensing and nervous control.
- The cardiac output is controlled by the sum of all local flows via a venous feed-back, but also requires further control of nervous origin.

Blood pressure management

- In general, blood pressure control is independent of local flow and cardiac output. If the pressure falls below 100 mm Hg, the nervous system intervenes with signals that:
  - Increase the strength of cardiac pumping
  - cause the contraction of the big veins, causing a greater blood flow to the heart
  - Generate general constriction of the arterioles as to have more blood in the arteries and therefore a higher arterial pressure.
  - Furthermore, on a longer time scale (hours and days) the kidney secretes hormones that control blood pressure and volume.

Stokes' viscous force

Sedimentation speed



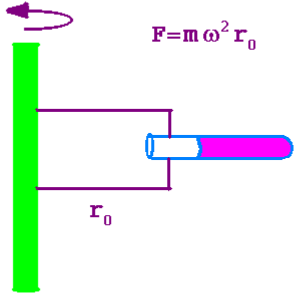
- A sphere of mass m falling in a viscous fluid η receives three forces:
- weight $\vec{F}_P = m\vec{g} = \rho_S V \vec{g}$
- Archimede $\vec{F}_A = m\vec{g} = \rho_L V \vec{g}$ (V sphere volume $V = 4/3\pi r^3$)
- Stokes viscous force$\vec{F}_S = -6\pi\eta r v$
- at equilibrium: $F_A + F_P = F_S$
- $6\pi\eta r v = \rho_S V\vec{g} - \rho_F V\vec{g} \Rightarrow$

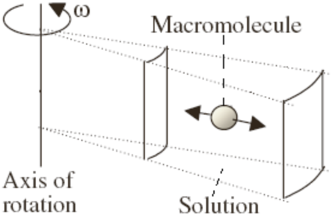
$$v = \frac{(\rho_S - \rho_F)V\vec{g}}{6\pi\eta r} = \frac{2}{9}\frac{(\rho_S - \rho_F)}{\eta} g\, r^2$$

Sedimentation: comments

- In a real fluid bodies that sink (or float) move with uniform rectilinear motion, after a transitory phase (in which they reach the constant velocity).
- Bodies of different density η and size r move with different final speed.
- this allows **separate** bodies with different properties while "sink" in the liquid: it is the sedimentation principle (eg to separate red, white cells and platelets from plasma).
- The same principle governs electrophoresis: bodies with different form and charge are driven by a constant electrical force (analogous to the weight force) in a viscous medium (eg gel) with the properties of a real fluid.

Centrifuge: $v_S = \frac{2r^2}{9}\frac{(\rho_S - \rho_F)}{\eta}\omega^2 R$





- To increase the sedimentation speed of small particles and/or to separate them from the solutions, the centrifuge is used
- this generates an acceleration $\omega^2 R$.
- Sedimentation velocity becomes a function of centrifugal speed (replacing $\omega^2 R$ a g)

$$v_S = \frac{2r^2}{9}\frac{(\rho_S - \rho_F)}{\eta}\omega^2 R$$

Gel electrophoresis

- The same phenomenon of sedimentation (based on weigth or centrifugal forces) is exploited in gel electrophoresis.
- In this case is a constant **electric field** E that acts on charged molecules (e.g. proteins in Western Blot, DNA in comet assay).
- Molecules with a different ratio between charge q and size (related to mass) move with different velocities:

$$F_A = F_{EL}$$

$$v = E \cdot \frac{q}{\beta}$$

- Particle velocity depends linearly on charge and inversely on its size (and medium viscosity, equal for all)

exercise

sedimentation of blood cells

- density of blood plasma is approximately $1025kg/m^3$
- density of blood cells circulating in the blood is approximately $1125kg/m^3$
- diameter of average immune cell is $10\mu m$
- blood viscosity $3.5 \cdot 10^{-3}$ pascal second