Electrostatic fields

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DIMES

Electric charge

Electrostatics

- Gravity is one of the fundamental forces present in nature, governing large-scale structure of the universe.
- The second class of fundamental forces is that of electromagnetic forces: they are responsible for the structure of matter (color, hardness, conductivity) as we know it.
- Atoms are made up of (negative) electrons and (positive) nuclei that interact by electric forces; the structure of matter depends on the electric forces exerted between atoms.
- Cause of these interactions is the **electric charge**. There are two types of electric charge, conventionally called **positive** and **negative**.

Electrostatics

- The existence of two types of charges causes the electromagnetic forces to be attractive or repulsive: charges of the same sign repel each other, while charges of opposite sign attract each other.
- Electrostatics deals with the forces between charged particles in stillness, or in motion, but with a velocity small enough to make magnetic interaction negligible.

Electric charge

- The charge of an electron is equal to and opposite to that of the proton: $|e| = 1.6 \times 10^{-19}$ Coulomb.
- Atoms have the same number of protons and electrons, Z, which defines the chemical species. They are therefore **neutral**.
- The number of neutrons N (equal to number of protons) defines the various **isotopes** of an atom.
- Atoms can become charged if their number of electrons changes (added or removed): charged atoms are called **Ions**.
- **Principle of conservation of the charge:** electric charges can not be created and destroyed, Motions of electric charges are defined as **electric currents**).
- The electrical charge of each element in Nature is a whole multiple of the elementary electric charge e.

Conductors and insulators

- Experimentally we observe that it is easy to generate movements of electric charges (electric currents) in metals.
- The explanation for this lies in the fact that the external electrons of the metals are practically free (**electron gas**) and therefore can move from one atom to another within the material.
- The metallic materials are therefore called **conductive**.
- Metals are good current conductors for the same reason that they are good conductors of heat, because of the mobility of their external electrons, called **conduction electrons**.

Conductors and insulators

- Other materials, such as glass or plastic, do not have equally mobile electrons, and therefore are not good conductors. These materials are then called **insulators**.
- There are also materials with intermediate characteristics: the **semiconductors**.
- The materials can then be classified on the basis of their ability to conduct electric currents.

Electrization by rubbing, contact and induction

- In a "planetary" representation of the atom, the electrons "orbit" around the nucleus and the outer ones are less bound, and can be removed more easily by ionizing the atom.
- Electrification for **rubbing** It is based on yielding and acquiring electrons if energy is supplied. The one who yields electrons becomes positively charged and is therefore able to attract negatively charged objects.
- Electrization for **contact**: if a negatively charged object is placed in contact with a neutral object, the electrons go from the negative object to the neutral object, which then will negatively charge.
- Electrification for **induction**: approaching a charged object to a conductive neutral object without touching it.

Electrization by induction

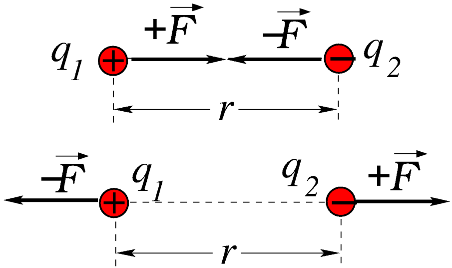
- If two neutral **conducting** spheres are contacted, and a negatively charged conductor approaches, the negative charge is placed as far as possible due to the electrostatic repulsion.
- If we separate the two spheres, one will remain negatively charged and the other positively.
- The phenomenon of induction is possible only for conductive materials, as it requires a macroscopic displacement of electrons inside the body.

Electric field

Coulomb's law

Coulomb (1785): $|\vec{F}|$ between q_1 and q_2 depends on their charges and on $1/r^2$:

$$|\vec{F}| = K\,\frac{|q_1 q_2|}{r^2}$$



K depends on the measurement unit, and on the medium between charges $K = N\cdot m^2/C^2$ In S.I. the unit is Coulomb (C). In vacuum: $K = 8.98\times 10^9\ \frac{N\cdot m^2}{C^2}$

The dielectric constant

Relative constant

$$K = \frac{1}{4\pi\varepsilon_0}$$

ε_0 **dielectric constant** in vacuum: $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$
Coulomb's law in vacuum

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} \quad (1)$$

In a medium:

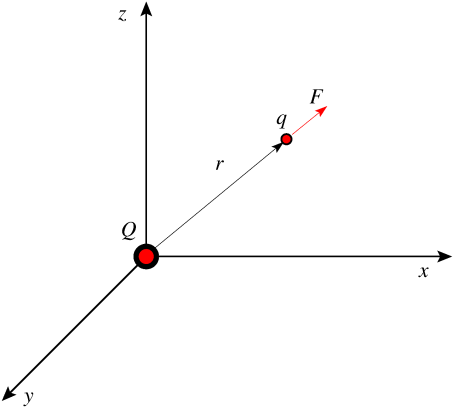
$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|q_1 q_2|}{r^2} \quad (2)$$

where $\epsilon_r > 1$ is a pure number **- relative dielectric constant -** depending on the medium.

some values of ϵ_r

Material	ε
Vacuum	1
Glass	5-10
Mica	3-6
Mylar	3.1
Neoprene	6.70
Plexiglas	3.40
Poliethilene	2.25
Teflon	2.1
Water	80.4
Glycerine	42.5
Ammoniac liquid(-78 C)	25
Benzene	2.284
Air(1 atm)	1.00059
Air(100 atm)	1.0548

Electric field



Force $\vec{F}$ depends on distance $\vec{r}$.
Repulsion if q and Q have same sign

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

$$\vec{F} = q\left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right]$$

Q generates an **electric field**

$$\vec{F} = q\vec{E}$$

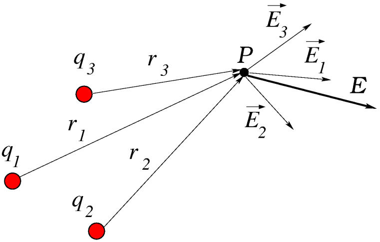
with:

$$\vec{E} = \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Superposition principle of electric fields

given n charges Q_1, Q_2, ..., Q_n in $\vec{r_1}$, $\vec{r_2}$, ...$\vec{r_n}$, the resulting electric field is the **vector sum** of the fields from each charge.

$$\vec{E}(\vec{r}) = \sum_{i=1}^{n} \vec{E}_i(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots + \vec{E}_n(\vec{r})$$



with

$$\vec{E}_i(\vec{r_i}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\vec{r_i}|^2} \quad i = 1, \ldots, n$$

and the force:

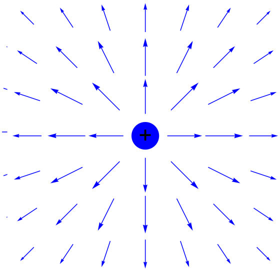
$$\vec{F}(\vec{r}) = q\vec{E}(\vec{r}).$$

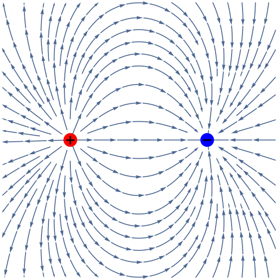
Electric field representation

- The electric fields can, as originally proposed by Faraday, be represented by the so-called **lines of force**.
- The lines of force always start from positive charges and end on negative ones without interruption and without intersecting.
- Moreover, the number of lines of force that come out of the positive charge or that enter into the negative one, depend on the intensity of the charge: where lines are thicker the field is more intense and vice versa.
- The field lines indicate the direction of the electric field: in each point the field is tangent to the line itself.
- The intensity of the electric field is proportional to the density of lines of force (number of lines crossing a unitary surface).

Field lines

a





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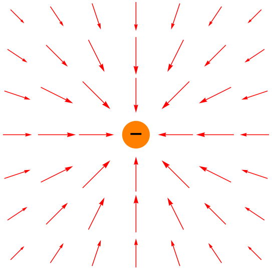
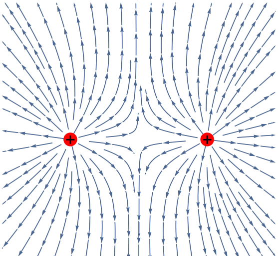
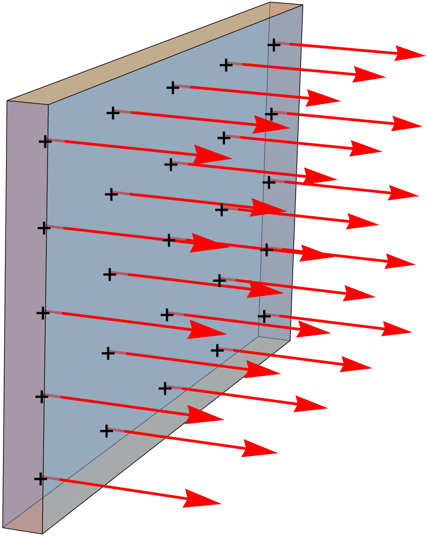




Uniformly charged plate



- One of the simplest charge configurations: distributing charges of the same sign uniformly, with a constant density, on a plane.
- If we show σ the charge density per unit of surface A:

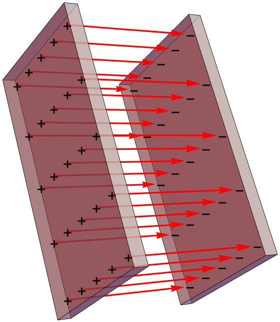
$$\sigma = \frac{q}{A}$$

- the electric field is:

$$E = \frac{q}{2\varepsilon_0 A} = \frac{\sigma}{2\epsilon_0}$$

- ε_0 vacuum dielectric constant.

paired charged plates



- The result for the single plate is useful for deriving the electric field between two uniformly loaded planes, one negative and the other positive.
- This device is called **capacitor**.
- In the capacitor the electric field is constant and perpendicular to the surface.
- For the calculation of the intensity of the field, applying Gauss's theorem, we obtain that:

$$E = \frac{q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

- σ surface charge density

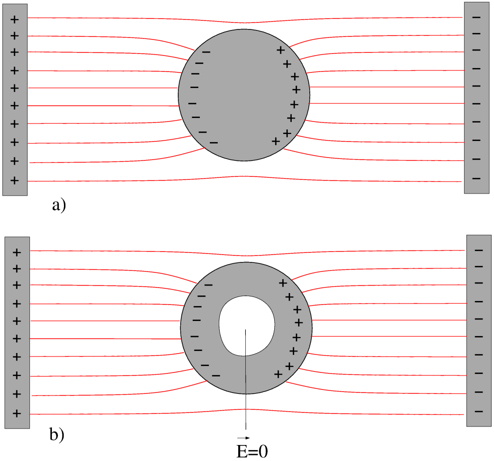
Conductors

Electric field in conductors

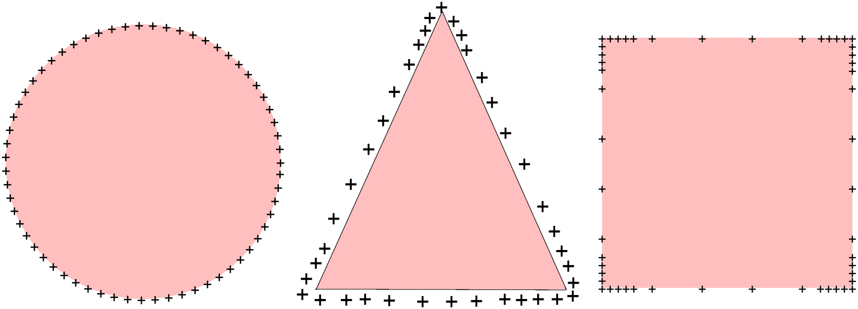
- In the conductors the electric charges are very mobile, and this influences the arrangement of the electric field both on the surface and inside the conductor.
- If electric charges of equal sign accumulate inside the conductor, a repulsive force begins to exert, which causes these charges to be arranged as far away from each other as possible, ie on the conductor's surface.
- Once the charges are all laid out on the surface, a situation of equilibrium is reached.
- **In a conductor the charges are arranged on the surface.**

Electric field in conductors

- When the charges are placed on the surface, inside the conductor, for symmetry reasons the electric field is null.
- **Within a conductor the electric field is null.**
- If we put a conductor in an electric field, a condition is created inside the conductor for which the electric field is null
- therefore, the lines of force of the electric field do not penetrate inside the conductor.
- This effect is the basis for the construction of **electrostatic shields**, called **Faraday cages** in honor of their creator, Michael Faraday.



(a) Conductor in a capacitor. The lines of force of the electric field do not penetrate inside the conductor (lines of force always perpendicular to the conductor).
(b) The electric field inside the conductor is zero.

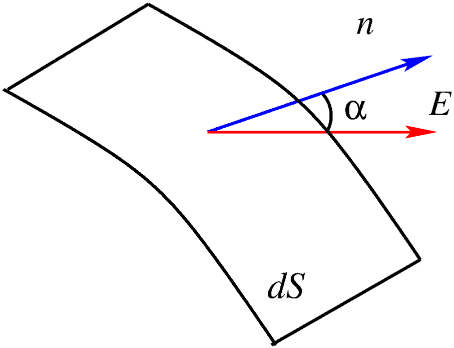


The electrical charges in the conductors are arranged on the surface and thicken in correspondence of the areas in which the conductor has some edges. *tip effect* of conductors.

Gauss

flux of $\vec{E}$

E (point charge) decreases with the square of the distance.



infinitesimal **flux** through dS is the scalar:

$$d\Phi(E) \equiv \vec{E} \cdot \vec{n} \cdot dS = E \cdot \cos\alpha \cdot dS$$

$\vec{n}$ versor orthogonal to surface

on a surface all infinitesimal fluxes sum up:

$$\Phi(E) = \int_S d\Phi = \int_S \vec{E} \cdot \vec{n} dS$$

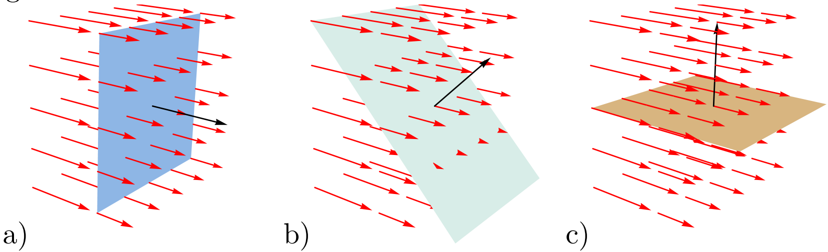
Closed surface $\Phi_S(E) = \oint_S \vec{E} \cdot \vec{n} dS$

Flux and surface orientation

For simple surfaces (squares) S and constant field:

$$\Phi_S(E) = S \cdot E \cdot \cos(\alpha).$$

geometric visualization:



a) maximal flux: $\cos(\alpha) = 1$

b) intermediate value: $\cos(\alpha) = \frac{\sqrt{2}}{2}$

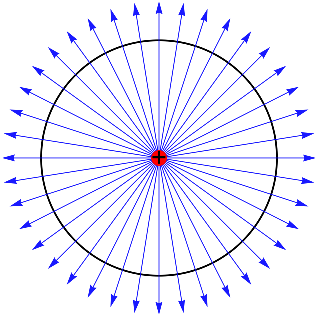
c) minimal flux: $\cos(\alpha) = 0$.

Gauss theorem

Gauss's theorem: the flow of the electric field through a closed surface is proportional to the net charge contained within the surface:

$$\Phi_S(E) = \frac{1}{\varepsilon_0} Q_{int}$$

Q_{int} algebric sum of internal charges.



for a single single charge:

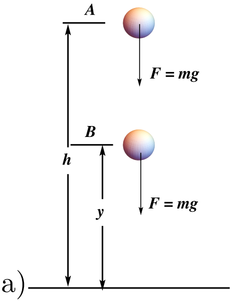
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

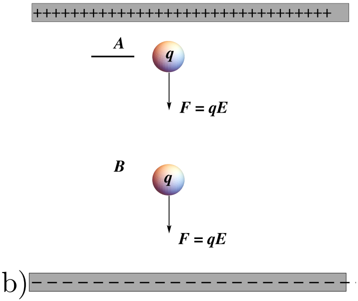
force lines $\perp$ to S and surface $4\pi r^2$:

$$\Phi = S \cdot E\cos(\alpha) = 4\pi \cdot r^2 \cdot \frac{q}{4\pi\varepsilon_0 r^2} = \frac{q}{\varepsilon_0}$$

potential ΔV

Work of the electric field, potential energy and electric potential

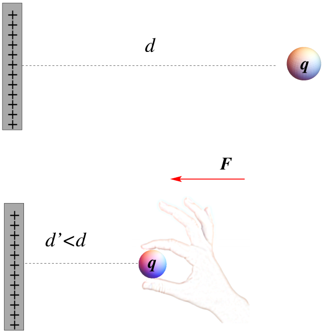




a) gravity mg acts on mass m. This force makes a work to move m from A to B.

b) Electric field E generates a force qE acting on q that makes a work to move q from A to B.

Work of the electric field, potential energy and electric potential



- The potential energy of the positive q charge is inversely proportional to the distance from the positive charge distribution.
- To bring the charge closer to the distribution you need to exert a F force and then perform a work that changes the potential energy of the charge.
- This work changes the potential energy of the loaded object as the work done to lift a serious change its potential energy.
- If the charged particle is then released, it gains an acceleration that pushes it away from the charged plate and its potential energy becomes kinetic.

Electrostatic potential energy

From the definition of work:

$$dL = \vec{F}(\vec{r}) \cdot d\vec{r}$$

with $\vec{F}(\vec{r}) = \vec{q}\vec{E}(\vec{r})$ si ha:

$$dL = q\vec{E}(\vec{r}) \cdot d\vec{r}$$

Coulomb's force is conservative and therefore its work does not depend on the path.
Also in this case we can define a function **electrostatic potential energy** whose variation is opposite to the mechanical work, and therefore, if we call U this function we have:

$$L_{AB} = -\Delta U$$

Electric potential

- In addition to the work of the electric force, the "work" of the electric field can also be defined (circuitation).
- Since the electric field is the electrical force on the test charge $E = F/q$, this work will be nothing but the mechanical work divided by the charge L/q.
- The potential energy on the charge defines a function whose variation gives the electric work

$$V = \frac{U}{q}$$

- Measurement unit is Volt (V) N/C.

$$\frac{L}{q} = \frac{F \cdot \Delta s}{q} = -\frac{\Delta U}{q} = -\Delta V$$

- The value of V in a point $\vec{r}$ has no physical meaning (known up to an arbitrary additive constant). What matters is the difference between two points.
- Positive charges are accelerated by regions with higher potential to those with lower potential, while negative charges have opposite behavior.
- Electron-Volt eV: is the potential energy acquired by an electron accelerated by a 1 Volt ddp:

$$1e \cdot V = 1.6 \times 10^{-19} C \times 1V = 1.6 \times 10^{-19} Joule$$

- A **equipotential surface** is a surface on which the electrical potential is constant at each point.
- For point charges the potential is equal if r is kept equal, so the equipotential surfaces are spheres centered in the charge.
- If an electric charge moves on an equipotential surface then the electric force performs null work.

Examples

Single positive charge.
Since $L = F \cdot r$ we get:

$$L = F \cdot r \ = \frac{qQ}{4\pi r^2} \cdot r = \frac{qQ}{4\pi r}$$

U is given by:

$$U = \frac{qQ}{4\pi r}$$

Electric potential $V = U/q$:

$$V = \frac{Q}{4\pi r}$$

Examples

capacitor has a constant E, force is $F = q \cdot E$.
The work is given by $F \cdot \Delta s$ (Δs distance between two plates).

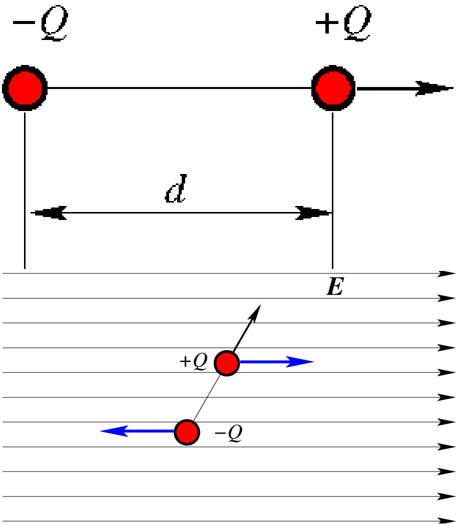
$$-L/q = \Delta V = -qE\Delta s/q$$

$$E = -\frac{\Delta V}{\Delta s}$$

This report says that the electric field is given by the **potential gradient**: variation of potential over distance.
The electric field is measured in V/m.

Capacitors

Electric dipole



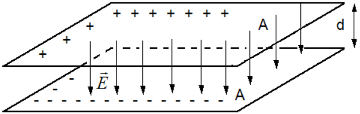
Vector $\vec{p}$, **electric dipole**:

$$|\vec{p}| = Q \cdot d$$

directed from $-Q$ to $+Q$. The electric dipole, in an electric field, orients itself so to minimize its potential energy $U = -\vec{p} \cdot \vec{E}$.

capacitor: $+q$ and $-q$.

Plain capacitor: parallel plates with surface A and distance d .



Uniform electric field inside, zero outside
Potential changes linearly:
$\Delta V = Ed$.

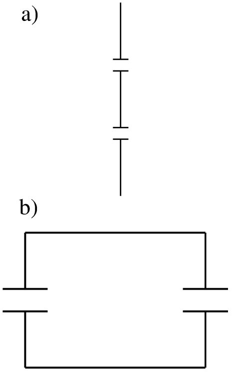
- V ddp between plates
- q charge on the plates
- "Capacity " $C = \frac{q}{V}$
- C depends only on capacitor shape.
- measure unit in SI C/V (Coulomb/Volt): Farad (F).

For flat capacitor: $C = \varepsilon \dfrac{A}{d}$

C increases with surface A, with the inverse of distance d and with the insulating power of the medium between plates.

Capacitors circuits

Capacitor is indicated as ||. Capacitors can be connected in series or in parallel



Two capacitors in series

$$\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{TOT} = \frac{C_1 C_2}{C_1 + C_2}$$

Two capacitors in parallel

$$C_{TOT} = C_1 + C_2$$

Energy of a capacitor

$$dL = V \cdot dq$$

since $V = q/C$

$$dL = \frac{q \cdot dq}{C}$$

L depends on q and C.

$$L = \int_0^Q \frac{qdq}{C} = \frac{1}{C}\int_0^Q qdq = \frac{1}{2}\frac{Q^2}{C}$$

Since $C = Q/V$:

$$L = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

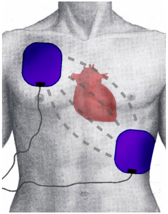
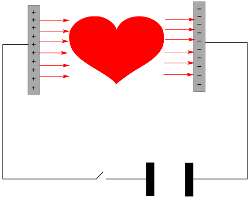
Applications

Biomedical applications

The applications in the biomedical field of electrostatic fields are innumerable, both in terms of applying electric fields to living organisms, and with regard to performing measurements of electrical activity produced (brain, muscle, and even cellular activity).

- Rest potential in electrically excitable and non-excitable cells. These potentials range from 20 mV to -70 mV
- Action potentials in excitable cells (non-Ohmic behavior)
- Measurements of electrical potentials
- Electroencephalography, Electromyography and Electrocardiography

During a heart attack, the heart produces irregular beats: **cardiac fibrillation**. A rapid and powerful electrical discharge that passes through the heart can remove the irregularity. Cardiac defibrillator: two planar electrodes connected to a capacitor generate an electric discharge (up to 1000 V) for a few milliseconds: after a few seconds the heart resumes its regular beat.

a)  b) 

a) The two electrodes must be applied on the chest to the sides of the heart;
b) simplified defibrillator scheme.

Many diagnostic techniques used in medicine are based on the fact that some points on the surface of the human body are at different electrical potentials. Typical potential differences are small, on the order of μV or a few hundred μ V.

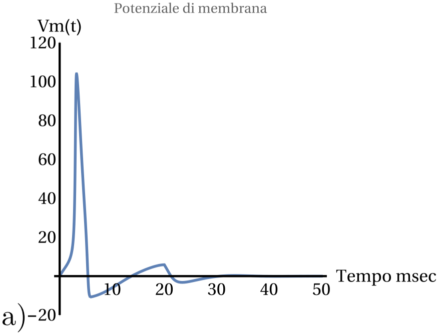
Signal	Amplitude
Electro-cardiogram (ECG)	1 mV
Electro-encephalogram (EEG)	10-100 μV
Electro-miogram (EMG)	300 μV
Electro-Oculogram	500μV
Membrane potential	10-100 mV

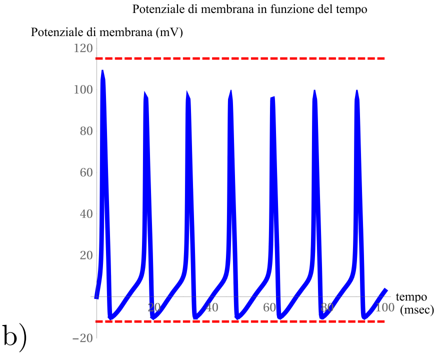
These electrical signals, appropriately amplified, can be detected by various devices such as those used in ECG, EEG, EMG, etc. These signals give information on the electrical activity of the cells involved, for example neuronal and muscular cells. The electrical activity of groups of these cells can also be detected on the surface of the human body.

Action potentials in cardiac muscle cells

Cells that respond to a current pulse in an active: excitable cells generating an action potential (neurons and muscle cells). The other cells are not electrically excitable, even if they have electrical activity.
A typical electrocardiographic pattern is obtained by electrodes placed on the surface of the human body. The difference in potential between two points changes when the heart beats and then repeats for each beat, forming a series of potential electrocardiograms (the shape of the electrocardiogram also depends on how the electrodes are arranged).
From an analysis of an ECG tracing, one can notice the characteristic regions, called P, Q, R, S and T that can be associated with specific phases of the cardiac cycle. Alterations in the ECG are a very important diagnostic tool for detecting arrhythmias, fibrillations or altered states of cardiac activity.

Action potentials in nervous cells

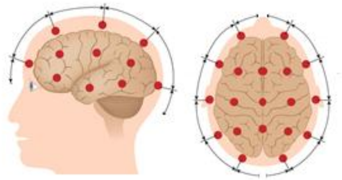




a) single action potential;
b) train of action potentials.
Train patterns determine intercellular nervous communication.

Electro-encephalogram

EEG is used to record the brain electrical activity. The device consists of a variable number of electrodes (from one to one hundred) that are placed in various regions of the skull. The various parts of the EEG are called waves or rhythms. The main rhythm of the brain is called rhythm (or wave) α and anomalies of the wave α can be linked to neurological disorders such as fatigue syndromes or sleep disorders.

a) 

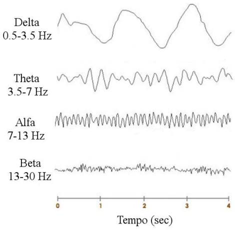
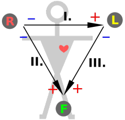
b) 

Figure: a) Typical electrode setting for EEG. b) EEG curves showing the most common rythms ($\alpha, \beta, \delta, \theta$).

Einthoven triangle for ECG



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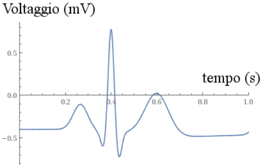

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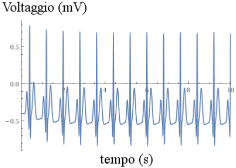

c)

a) Einthoven triangle for ECG.
b) Single heart beat.
c) Train of pulses associated to heart beat.

Exercise

an accelerating proton

- In a vacuum a proton (charge=+e, mass=1.67 x 10^{-27} kg) is moving parallel to a uniform electric field
- $e = -1.6 \cdot 10^{-19}$
- the field is direct along the x axis (the same direction of the motion of the proton)
- the proton starts with a velocity of 0 m/s and accelerates in the same direction as the electric field
- the electric field has a value of 2.3 x 10^3 N/C
- find the velocity of the proton then the displacement is 2.0 mm from the starting point

an accelerating proton - solution

- since we know the initial velocity and the displacement, we can determine the final velocity from the equations of kinematics, provided the proton's acceleration can be found
- the acceleration is given by Newton's second law as the net force acting on the proton divided by its mass
- the net force is the electrostatic force, since the proton is moving in an electric field
- the electrostatic force depends on the proton's charge and the electric field, both of which are known

$$a = \frac{F}{m} = \frac{eE}{m} \Rightarrow \Delta x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

$$t^2 = \frac{2\Delta x}{a}$$

$$v_x = v_0 + a \cdot t = \sqrt{2a\Delta x} = 2.10 \cdot 10^4$$