Electric Currents

Enrico Giampieri

DIMES

currents

battery and the study of electricity

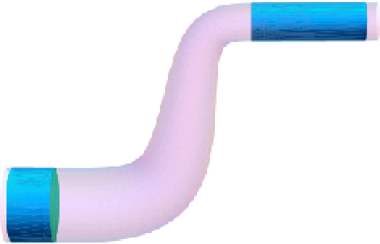
- the first experimental observation of electric currents are from the Luigi Galvani experiments in the XVIII century with frogs, performed here in Bologna
- Alessandro Volta took inspiration from Galvani's studies for developing the Voltaic battery
- with the battery it became possible to study the phenomenon of electric current
- electric current is the movement of charged particles in a conductive medium.
- in metals we have the motion of electrons
- in biological tissues we typically have motion of charged ions in solution, such as Na, Cl, K, Ca

Electric current in a battery

- if two points of a conductive medium are at two different electric potential values, we have a different potential energy of the electrons
- this generate a net force (called **Electromotive Force,** emf) that moves the electrons
- they start to move from the point of high energy to the low energy ones
- the electron accumulate in the low electric potential point (the positivly charged side), neutralizing the positive charge and reducing the potential difference
- given enough time, the voltage difference goes to 0, and the current stops.

Water analogy

- Electrical current can be imagined as the flow of water along pipes at different levels
- higher electric potential are equivalent to higher differences in height.
- the water amount would be the charged particles
- the water flux is the equivalent of the electric current



Electric current definition

- Electrical charges move inside a wire and crossing an imaginary surface that is perpendicular to their motion.
- This flow of charge is known as an electric current.
- The electric current I is defined as the amount of charge per unit time that crosses the imaginary surface
- If the rate is constant, the current is:

$$I = \frac{\Delta q}{\Delta t}$$

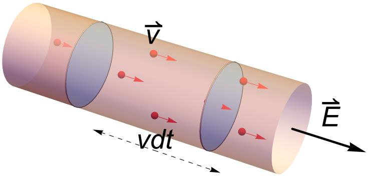
Electric current definition

- If the rate of flow is not constant, then the Eq. gives the average current.
- Since the units for charge and time are the coulomb (C) and the second (s), the SI unit for current is a coulomb per second (C/s).
- One coulomb per second is referred to as an ampere (A).
- If the charges move around a circuit in the same direction at all times, the current is said to be direct current (dc), which is the kind produced by batteries.
- In contrast, the current is said to be alternating current (ac) when the charges move first one way and then the opposite way, changing direction from moment to moment.

current density

- The amount of current that goes through a certain ares in an amount of time is called **current density**
- For a wire of area A

$$j = \frac{i}{A}$$



current density - microscopic model

- Imagine a conductor containing ρ free electron (with charge e) per unit of volume
- these electrons are moving at speed v
- in a time interval dt they move a distance $v \cdot dt$
- We want to count those that pass an imaginary area A in the interval dt
- only those at a distance less than $v \cdot dt$, so the volume $V = (v \cdot dt) \cdot A$
- the number of electron the passes per unit of time are $N = \rho V = \rho \cdot (v \cdot dt) \cdot A$
- the current is going to be the charge of the electron times the number of electrons divided by the unit of time

$$I = dQ/dt = e \cdot \rho \cdot (v \cdot dt) \cdot A/dt = e \cdot \rho \cdot v \cdot A$$

$$j = \frac{I}{A} = \frac{e \cdot \rho \cdot v \cdot A}{A} = \rho e v$$

Ohm's Laws

electric resistance

- the relationship between electric current and efm is similar to the relationship between pressure and fluid flow in a pipe
- increasing the efm (or the pressure) one can observe an increase in electric current (or fluid flow)
- the relationship is linear: the current i is proportional to the voltage V
- Thus, a voltage of 12 V leads to twice as much current as a voltage of 6 V, when each is connected to the same circuit.
- Longer and narrower pipes offer higher resistance to the moving water and lead to smaller flow rates for a given pump pressure.
- A similar situation exists in electric circuits, and to deal with it we introduce the concept of electrical resistance.

First Ohm's Law

- we can introduce the idea of **electric resistance** of a medium to describe this relationship
- the resistance of a conductive object is given by the relationship between the voltage V and the current I
- The resistance R is defined as the ratio of the voltage V applied across a piece of material to the current I through the material, or

$$R = \frac{V}{I}$$

- When only a small current results from a large voltage, there is a high resistance to the moving charge.
- Resistance is measured in **Ohm** (Ω)

First Ohm's Law

- For many materials (e.g., metals), the ratio V/I is the same for a given piece of material over a wide range of voltages and currents.
- In such a case, the resistance is a constant.
- Then, the relation $R = V/I$ is referred to as Ohm's law, after the German physicist Georg Simon Ohm (1789?1854), who discovered it
- it is also possible to define the conductance G of an object as the inverse of the resistance $G = R^{-1} \Rightarrow G = \frac{I}{V}$

Second Ohm's Law - Resistance and resistivity

- For a wide range of materials, the resistance of a piece of material of length L and cross-sectional area A is

$$R = \rho \frac{L}{A}$$

- Where ρ is a proportionality constant known as the **resistivity of the material**.
- It can be seen that the unit for resistivity is the ohm meter (m)

material resistivity and temperature

- Material resistivity depends on the temperature of the medium
- the resistivity of the filament of an incandescence lightbulb is much higher when it is turned on than when is off
- in metals the resistivity increases with temperature, in semiconductors is often the opposite

Material	Resistivity ρ $(\Omega \cdot m)$
Steel	1.8×10^{-7}
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Tungsten	5.5×10^{-8}
Grafite	8×10^{-6}

for many materials:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where ρ and ρ_0 are the resistivity at temperature T and T_0 while α depends on the material and is positive when the resistivity increases with temperature

resistivity of human tissue

Tissue	Resistivity ρ (Ω m)
spinal fluid	0.650
Plasma	0.7
Whole blood	1.6
Skeletal muscle	3-18
Liver	7
Lungs	8-17
Neural tissue	2-7
Fat tissue	20
Bone	40
Skin	$10^5 - 10^7$

Joule's effect

- due to the electrical resistance the regime speed of electrons in the medium in constant
- the energy that they gain from the Electromotive force is dissipated by hitting the material's atoms and is dispersed as heat
- this is called Joule effect
- The energy of the charge dq goes down by $V dq$ and due to the first principle of thermodynamics this should be the amount of heat dQ released

$$dQ = V dq$$

- the heat produced per unit of time is the dissipated power P

$$P = \frac{dQ}{dt} = V\frac{dq}{dt} = VI$$

Joule's effect

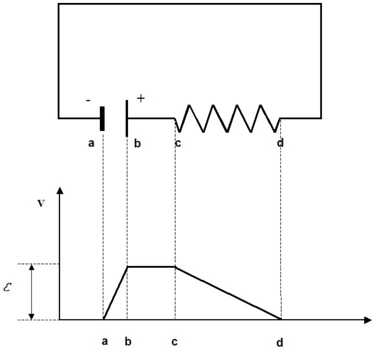
- it can also be rewritten, using the first Ohm's law, as:

$$P = VI = I^2R = \frac{V^2}{R}$$

- if the tension V is constant, the power P is proportional to the resistance R
- the unit of measurements is the Watt

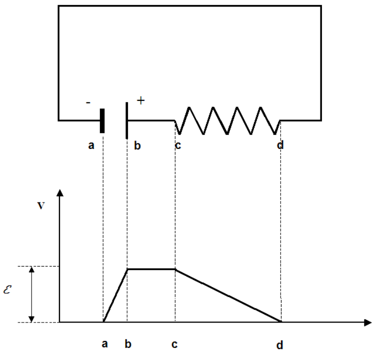
circuits

Electric circuits



- in a circuit I have a potential generator that create a voltage $\mathcal{E}$
- I can describe the wire as having no resistance and letting the current flow without obstacles
- I represent the overall resistance of the circuit with a single resistor with resistance R

Electric circuits

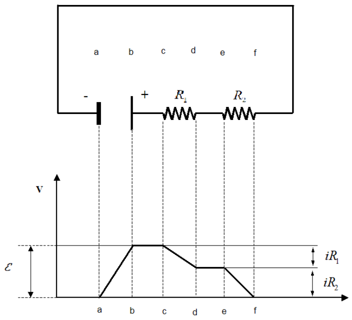


- from Ohm's law the current becomes:

$$\mathcal{E} = i \cdot R$$

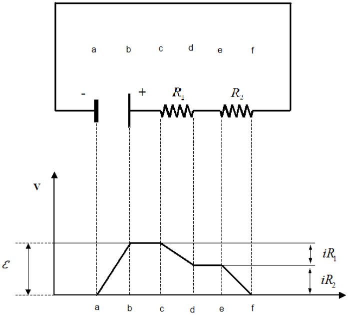
- all the potential generated by the generator should be lost after the resistor
- I can study how the potential energy of the electron becomes point by point

Resistors in Series



- in a single wire the current has to be the same everywhere
- each resistor reduces the voltage by the amount predicted by the Ohm's law
- the total drop should be equal to the energy provided by the generator

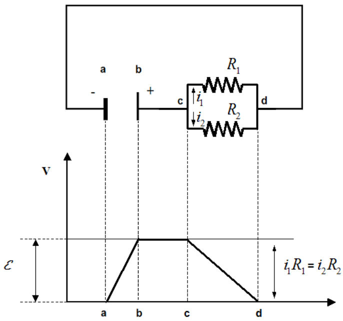
Resistors in Series



$$\mathcal{E} = i \cdot R = i \cdot R_1 + i \cdot R_2$$
$$= i \cdot (R_1 + R_2)$$

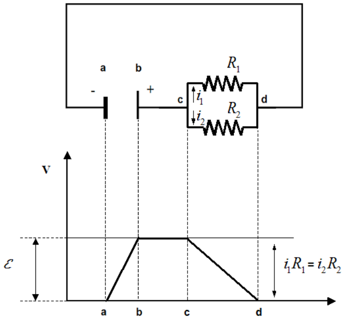
$$R = R_1 + R_2$$

Resistors in Parallel



- when a wire split, the current has to be divided between the branches, but the sum must give the same total current
- the drop of tension should be the same in the two branches
- otherwise when they join back they would have two potential energies in the same point at the same time

Resistors in Parallel



$$i = i_1 + i_2$$

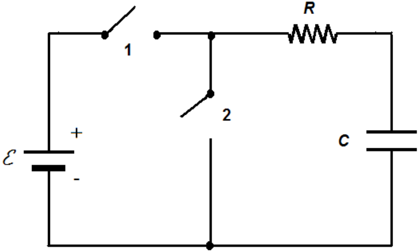
$$i_1 R_1 = i_2 R_2 = \mathcal{E}$$

$$i_1 = \mathcal{E}/R_1 \text{ e } i_2 = \mathcal{E}/R_2$$

$$i = i_1 + i_2 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

RC circuits - charge and discharge



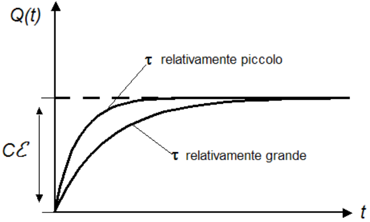
$\tau = RC$ **time constant.**

charge:

$$V(t) = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

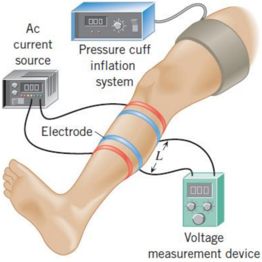
discharge:

$$V(t) = \mathcal{E}e^{-\frac{t}{RC}}$$



Applications

impedance plethysmography



- Diagnosis of blood clotting in the veins (deep venous thrombosis) near the knee.
- A pressure cuff, like that used in blood pressure measurements, is placed around the midthigh, 4 electrodes are attached around the calf.
- The two outer electrodes are connected to a source that supplies a small amount of ac current.
- The two inner electrodes are separated by a distance L, and the voltage between them is measured.

impedance plethysmography

- The voltage divided by the current gives the resistance.
- Resistance can be related to the volume V_{calf} of the calf between the inner electrodes.
- The volume is the product of the length L and the calf's cross-sectional area A, or $V_{calf} = LA$.
- Solving for A and substituting in the previous Eq shows that

$$R = \rho \frac{L}{A} = \rho \frac{L}{V_{calf}/L} = \rho \frac{L^2}{V_{calf}}$$

impedance plethysmography

- Thus, resistance is inversely proportional to volume, a fact that is exploited in diagnosing deep venous thrombosis.
- Blood flows from the heart into the calf through arteries in the leg and returns through the system of veins.
- The pressure cuff is inflated to the point where it cuts off the venous flow but does not alter the arterial flow.
- As a result, more blood enters than leaves the calf.
- Therefore, the volume of the calf increases, and the electrical resistance decreases.
- When the cuff pressure is removed, the volume returns to a normal value, and so does the electrical resistance.
- With healthy (unclotted) veins, there is a rapid return to normal values.
- A slow return, however, reveals the presence of clotting.

Electrodermal activity

- It is possible to measure skin conductivity to small currents
- this conductivity increases with the release of sweat and increased blood perfusion
- most stimuli that generate sympathetic response increase the sweat production
- this can be used as a measurement of psychological or physiological arousal
- this in turn can be used to measure anxiety, stress or pain responses to various stimuly
- is a common method for investigating human psychophysiological phenomena

Cardiac pacemaker

- the cardiac pacemakers are based on an RC circuit that controls the timing of the voltaic impulses sent to the hearth
- this impulses are transmitted by electrodes that can be either internal or external (implated with surgery)
- they contains a small capacitor, which builds up charge over time
- when the charge goes over a certain value, a trigger happens to discharge the capacitor and send a pulse
- the capacitor is then recharged and the cycle repeat at regular intervals
- the time interval between pulses is driven by the RC constant and is set to be around 1 second, similarly to the regular cardiac cycle

Biological effects of electric currents

- electrical shocks can be damaging and even lethal
- they can cause sever burns, cardiac fibrillation, muscle and nervous tissue damage
- the severity of the effects depends on the current intensity and type (continuous or alternate)
- in general continuous current is less dangerous, as it does not interfere with regular tissue mechanism and requires a lot of power to damage the tissue by Joule's effect
- alternate current is more dangerous, as it can interfere with the functioning of nervous tissues and muscles, including the hearth

Biological effects of electric currents

- the effects are usually divided in micro and macro shocks
- macroshocks happen when a significant amount of current goes trhought the body including the hearth
- microshocks happen when the electric current is weaker and more localized
- currents under the milliampere are not dangerous and usually lead to skin crawling sensations
- currents over 10 mA can cause muscle spasms and loss of muscle control, but are rarely lethal
- currents above 0.5 A can generate cardiac fibrilation and can thus lead to death
- a common danger is the muscle spasm of the hand leading to extended holding of the source of current: always check with the back of your hand!

Exercise

A condenser

- A flat condenser has the armatures of area 8 cm^2 spaced by a distance of 2 mm.
- in between the armatures there is empty space
- If a potential difference of 20 Volts is applied to the armatures, calculate:
  1. The electric field between the armatures;
  2. the surface charge density;
  3. the capacitance of the capacitor;
  4. the charge on each armature;
- if a resistor with resistance $R = 10^6\,\Omega$ connects the armatures, calculate:
  5. the initial current when connected
  6. the time constant of the circuit

A condenser - solution

1. definition of electric field: $E = V/d = \frac{20}{2 \cdot 10^{-3}} = 10^4 V/m$
2. for two parallel plates:

$$E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = E\epsilon_0 = 10^4 \cdot 8.85 \cdot 10^{-12} = 8.85 \cdot 10^{-8} C/m^2$$

3. for flat capacitor:

$$C = \varepsilon_0 \frac{A}{d} = (8.85 \cdot 10^{-12}) \cdot (8 \cdot 10^{-4}) \cdot (2 \cdot 10^{-3})^{-1} = 3.54 \cdot 10^{-12} F$$

4. various solutions:
   - using the definition of the surface density $Q = \sigma \cdot A = 8.85 \cdot 10^{-8} \cdot 8 \cdot 10^{-4} = 70.8 \cdot 10^{-12}$
   - using the capacitor definition $Q = C \cdot V = 3.54 \cdot 10^{-12} \cdot 20 = 70.8 \cdot 10^{-12}$
5. the fully charged capacitor applies the whole potential and we can use the Ohm's first law: $I = V/R = 20/(10^6) = 2 \cdot 10^{-5} A$
6. this is defined as $\tau = RC = 10^6 \cdot 3.54 \cdot 10^{-12} = 3.54 \cdot 10^{-6} s$