

Magnetic Field and EM waves

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DIMES

magnetism

Magnetism

- Magnetic phenomena have always attracted the interest of man because of their effects, and have for some time maintained an aura of mystery for their apparent inexplicability.
- The term *magnetism* derives from the name of an ancient Greek city, *Magnesia*, where, over 2000 years ago, the first magnetic minerals characterized by the property to attract iron were observed.
- These minerals were then called *calamite*, and were first used as compasses by the Chinese, starting from about 1200 thanks to their ability to indicate a fixed direction, from which their use especially for navigation.

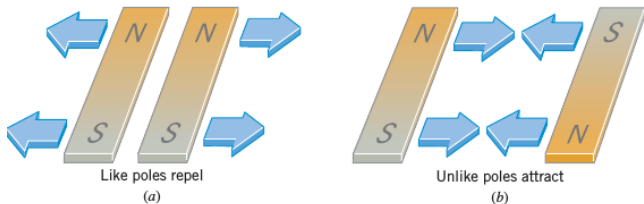
Magnetism

- Around 1500 William Gilbert was able to produce artificial magnets through the contact of pieces of iron against the magnets
- was the first to suggest that the compasses pointed to the North due to the presence of the Earth's magnetic field and that therefore the Earth itself was a giant magnet.
- Subsequently, it was observed that between two magnetic poles some forces were established whose intensity depended on the distance between them, which was then confirmed by Coulomb.

Magnetism

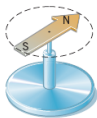
- Electricity and magnetism also appeared as independent phenomena
- in about 1820 a Danish physicist, Hans Christian Oersted, showed how electric currents were able to exert forces on permanent magnets
- therefore proposed the existence of a link between electrical and magnetic phenomena.
- This link was then definitively explained by Andre Marie Ampere, who showed that electric currents are the basis of all magnetic phenomena.

Magnet poles



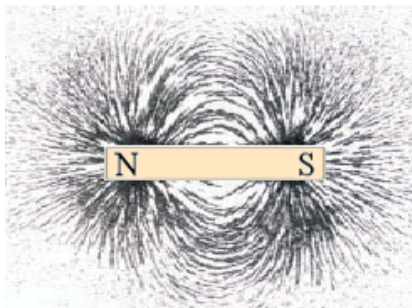
- The permanent magnets have two poles, "north" and "south". Poles of the same type (a) repel each other, opposite poles (b) attract each other.
- While it is possible to separate positive and negative charges, magnetic poles are always coupled.
- Even dividing a magnet into several parts repeatedly, the two poles always remain united.
- **There is no free monopole**

Compass



- The compass needle is a permanent magnet that has a magnetic north pole N at one end and a south pole S at the opposite end.
- In the region around the magnet we can hypothesize the presence of a field, the magnetic field, able to produce a force in the presence of another magnet.
- the compass can be used to study the orientation of the field lines of the magnetic field

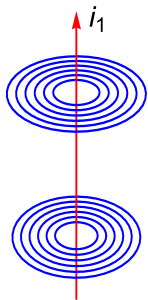
permanent magnet fields



- Magnetic field lines around a magnet
- the field made visible by iron filings
- Magnetic field lines in a magnet with facing poles
- **Magnetic field lines are always closed**

Magnetic Field

Field of a single wire



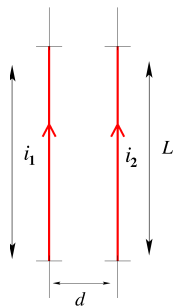
- A rectilinear wire generates a magnetic field with cylindrical symmetry, with intensity inversely proportional to the distance from the wire

$$B = \frac{\mu_0 i}{2\pi r}$$

- the unit of measurement of the magnetic field B in the S.I. It's $N/A \cdot m$, called Tesla.
- μ_0 is called the **vacuum permeability** and have a role in magnetic fields equivalent to the ϵ_0 for electric fields, $\mu_0 = 1.26 \cdot 10^{-6} N/A^2$ (Newton/Ampere squared)
- This field manifests itself through a force that is exerted on **moving electric charges** (single, currents).

Fields of parallel wires

- Two parallel wires, long L and d far away, with currents i_1 and i_2 :



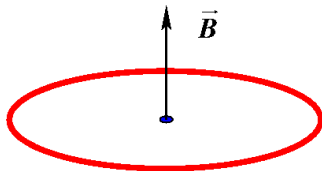
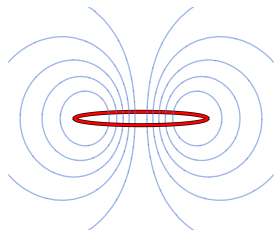
$$F_{12} = \frac{\mu_0}{2\pi} \cdot \frac{i_1 i_2 L}{d}$$

- using the **magnetic field** generated by wire 1:

$$F_{12} = \left(\frac{\mu_0 i_1}{2\pi d} \right) i_2 L = B \cdot i_2 L$$

- i_1 creates a field whose intensity depends on the current and the distance d from it.
- A second current i_2 is affected by a force due to the presence of the field of **magnetic induction** B .

Field of a spire



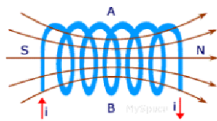
The magnetic field in the center of the loop has an intensity:

$$B = \mu_0 \frac{i}{2r}$$

where i is the circulating current and r the radius.

Solenoids

- Solenoid generates a **constant** magnetic field inside, directed along its axis, while the external magnetic field is approximatively zero.



$$B = \mu_0 n i$$

- $n = N/l$ is the number of loops per unit length and i is the current.
- Solenoids are used in Medicine to generate strong magnetic fields in Nuclear Magnetic Resonance

Magnetic force on a moving charge

- When we place an electric charge in a magnetic field, a force is generated acting on the charge, provided that:
 - the charge is in motion
 - the velocity of the particle has a component perpendicular to the magnetic field
- If the charge moves perpendicular to the magnetic field then the force acting on it is maximal, whereas if the velocity of the particle forms an angle θ with the force lines of the magnetic field the force is less.
- The force is perpendicular to the direction of the magnetic field and to the velocity of the particle (ie the plane containing \vec{v} and \vec{B}),
- so if the particle deviates its direction, also magnetic force changes direction
- **NOTE: \vec{B} and \vec{F} are always perpendicular to each other**

Lorentz's force

The force acting on q moving with speed \vec{v} in a field \vec{B} :

$$\vec{F} = q\vec{v} \times \vec{B}$$

\vec{F} : **Lorentz Force**, "×" vector product. The modulus of F :

$$F = q \cdot v \cdot B \cdot \sin(\theta)$$

If $\theta = \pi/2$ the particole moves orthogonal to the field

$$F = q \cdot v \cdot B$$

A particle with a charge 1 C moving at 1 m/s orthogonal to a magnetic field of 1 Tesla receives a force of 1 N .

rotation due to the Lorentz's force

- Equating Lorentz force to centrifugal force:

$$m \frac{v^2}{R} = qvB$$

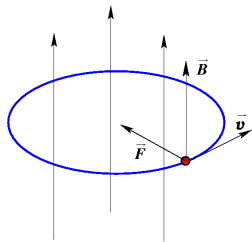
- the radius is :

$$R = \frac{mv}{qB}$$

- Pulsation ω is proportional to charge/mass ratio:

$$\omega = \frac{2\pi}{T} = \frac{v}{R} = \frac{q}{m}B$$

- Radius is inversely proportional to B , pulsation is directly proportional to B .

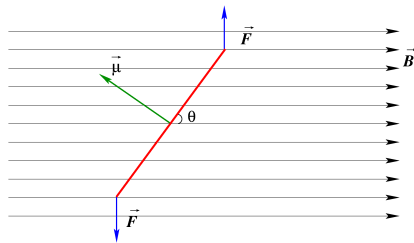


Magnetic momentum

Magnetic momentum

In the presence of a magnetic dipole p with a pole distance l , or a current i in a loop with area S a **magnetic moment** μ is created:

$$\vec{\mu} = p \cdot \vec{l} = i \cdot S \cdot \hat{u}$$



In a magnetic field \vec{B} a torque \vec{M} is generated:

$$\vec{M} = \vec{\mu} \times \vec{B}$$

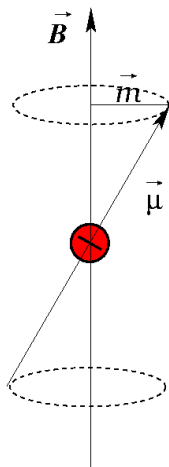
Response of materials to an external field B_0

- different materials have different reactions when exposed to an external magnetic field.
- materials **weakly rejected** from the external magnetic field (such as water, silver, copper) are called **diamagnetic**
- materials **weakly attracted** by the external magnetic field (aluminum, platinum) are called **paramagnetic**
- materials **strongly attracted** from the field (such as iron, nickel, cobalt) are called **ferromagnetic**
- The explanation is in the different atomic magnetic moments of spin.

Response of materials to an external field B_0

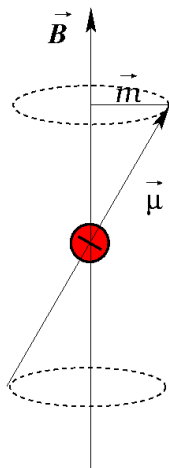
- **Diamagnetic** have null atomic magnetic moments (there are no unpaired electrons): an external field induces a weak magnetic field in the material that results opposite to B_0 (quantum effect).
- **Paramagnetic** have atomic magnetic moments different from zero (they possess unpaired electrons) but randomly oriented. An external magnetic field aligns them, and a weak magnetic field is generated in the material.
- **Ferromagnetic** have non-zero atomic magnetic moments, but magnetic moments are organized into domains containing up to $10^{16} - 10^{19}$ atoms (some *mm*) and originate a macroscopic magnetic field, as in natural magnets.

Larmor's frequency



- A magnetic moment $\vec{\mu}$ in a uniform field \vec{B} receives a momentum \vec{M} which tends to align it with the field \vec{B} .
- If a magnetic moment $\vec{\mu}$ is immersed in a magnetic field \vec{B} we have the *precession* phenomenon of $\vec{\mu}$ around the axis \vec{B}
- this is due to the torque \vec{M} exercised by \vec{B} on $\vec{\mu}$ ($\vec{M} = \vec{\mu} \times \vec{B}$).

Larmor's frequency



- This motion, has a characteristic frequency, called **Larmor's frequency**:

$$\omega = \gamma \cdot B$$

- γ depends on the charge and mass ratio of the electron:
- $\gamma = e/2m$ which is called the electromagnetic giromagnetic ratio.
- The Larmor's frequency is at the base of the phenomenon of Nuclear Magnetic Resonance.

Nuclear Magnetic Resonance

- Static magnetic fields (1 - 7 T) are produced from superconducting solenoids.
- Some atoms (eg H, P, C, Na), when subjected to these fields, behave like transmitting antennas and send electromagnetic signals in the radio-waves range.
- Because of the precession of Larmor, these atoms receive energy which they then emit in the form of photons with specific frequencies.
- If the intensity of the field varies from point to point, different regions of the observed sample (subject to fields of different value) will emit photons with different frequencies.
- So from the frequency of the photons received we can locate the source (the part of the organism) that generated them.

NMR applications

- Spectroscopic techniques for the identification of biochemical compounds in organs and tissues of interest
- Imaging Techniques (Magnetic Resonance Imaging, MRI).
- diffusion in tissues (brain fibers imaging)

BOLD MRI

- Through appropriate **reconstruction** algorithms, starting from the received signals, images can be obtained based on the different distribution of hydrogen in the tissues.
- An NMR technique called BOLD (Blood Oxygenation Level Dependent), is based on the different magnetic properties of oxygenated and deoxygenated hemoglobin (deoxyhemoglobin) and is used to obtain information on in vivo metabolic activity
- brain activity = oxygen consumption

BOLD MRI

- A different percentage of oxygenated hemoglobin can provide information on the consumption of oxygen (and therefore indirectly of metabolic activity) in a certain district.
- In particular, with this technique, in vivo observations of brain activity are performed to identify pathologies or to characterize which regions are activated in response to certain external stimuli.
- The BOLD technique is at the base of all the so-called “functional” NMR methods that allow not only the anatomical characteristics of a sample to be observed, but also the activity (in this metabolic case) that takes place in it.

Induction

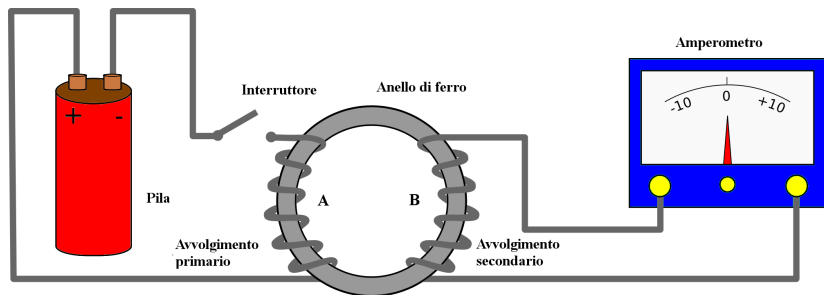
Electromagnetism

- 1830: Michael Faraday (UK) and Joseph Henry (USA) discover that a magnetic field varying in time can induce an electromotive force inside a circuit (an electric current).
- Therefore, magnetic and electric fields are not separated entities, but belong to a unified force type: electromagnetic forces.
- This observation posed the basis for the conversion of mechanical energy into electric energy.

Electromagnetism

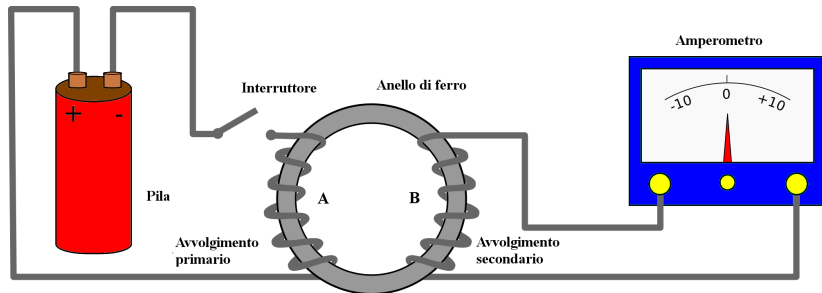
- Later, J. C. Maxwell formalized these electromagnetic phenomena with a mathematical formalism, that led to hypothesize the existence of electromagnetic waves, i.e. the remote propagation of electromagnetic energy.
- In this way, also optics was framed in the field of electromagnetic wave propagation.
- Electromagnetic phenomena intervene in the generation of electromagnetic waves and in the definition of the wave nature of light and its propagation.

First Faraday experiment



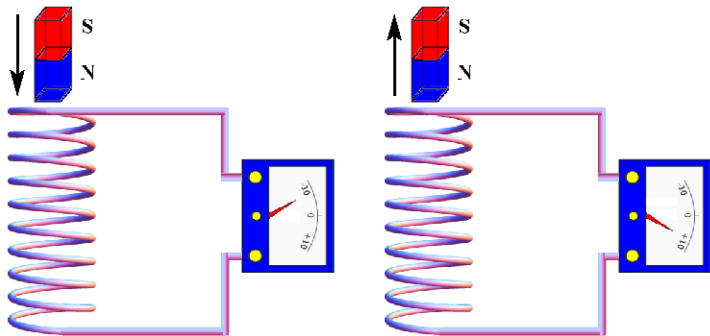
- a set of turns, A, is connected to a stack, which produces a current in the **primary** circuit.
- This current generates a magnetic field (intensified by the iron ring).
- The **secondary** circuit B is connected to an amperometer which measures the electric current.

First Faraday experiment



- With a steady-state current in A no electrical current is measured in B.
- Current in B is observed only while the primary circuit is opened or closed via the switch.
- Furthermore, the observed current in B has opposite direction to that of the primary circuit A.
- We have *induced current* only if the magnetic field changes over time

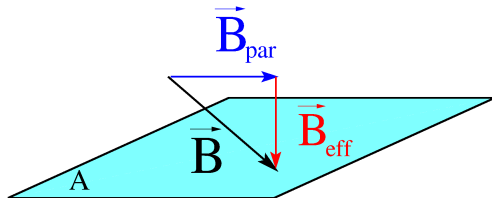
Second Faraday experiment



If a *Solenoid* is approached by a moving permanent magnet, an electric current is measured.

Effective magnetic field

- The induced electromotive force in a *solenoid* is proportional to the number of turns and the speed of variation of the concatenated magnetic field
- the term "concatenated" indicates that the lines of force of the magnetic field pass through the circuit.
- The effective component of magnetic field \vec{B} is that perpendicular to the surface A
- therefore depends on the θ angle that the field forms with the line perpendicular to the surface.



flux of the magnetic field

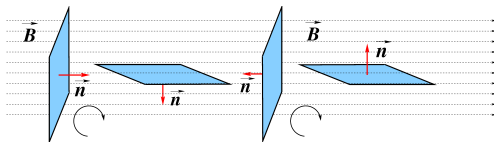
- The **flux** of the magnetic field through the surface A can be expressed as:

$$\Phi_A(\vec{B}) = B_{eff} \cdot A = B \cdot A \cdot \cos(\theta) = \vec{B} \cdot \vec{A}$$

- The flow of the magnetic field through the surface A quantifies the notion of lines of force concatenated to the circuit.
- Faraday's law can therefore be expressed by the following formula:

$$emf = -\frac{\Delta\Phi_A(\vec{B})}{\Delta t}$$

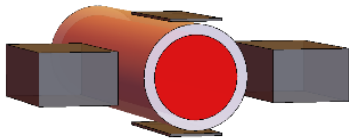
Flux change



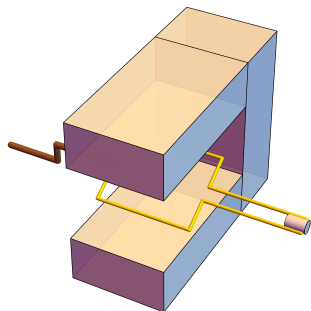
- Variation of the magnetic field flux with the position of the circuit.
- The circuit is progressively rotated, the angle formed between the normal vector \vec{n} at the surface and the magnetic field strength lines \vec{B} varies accordingly
- therefore also the flow $\Phi_A(\vec{B})$ varies between a minimum value and a maximum value
- Flux can change in three ways:
 - variation of magnetic field B intensity
 - variation of the concatenated surface A with the B field
 - variation of the angle between field B and surface A

Measurement of blood velocity

- The velocity of blood flow can be measured by the electromagnetic properties of the blood.
- The blood contains ions in solution, therefore it behaves like a conductor, so its motion induces an electromotive force that can be measured.
- Blood flow F , if placed inside a magnetic field, generates a *emf* proportional to its velocity.



Sinusoidal current



- The change in B flux is determined by the varying angle between B and the surface of the circuit.
- If the circuit rotates with uniform circular motion at angular velocity ω , the flow value will oscillate over time in a sinusoidal manner:

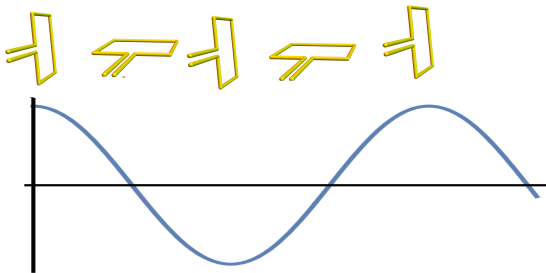
$$\Phi_B(t) = B \cdot S \cdot \cos(\omega t)$$

- this will induce a sinusoidal *alternating current*:

$$i(t) = \omega/R \cdot B \cdot S \cdot \sin(\omega t)$$

Sinusoidal current

Value of the intensity of the induced current according to the orientation of the turn. Note that the current has an intensity with a sinusoidal pattern that it is alternately greater and smaller than zero.



EM waves

Electromagnetic waves

- The greatest success of electromagnetic theory was the prediction of the existence of electromagnetic waves, and the hypothesis that such waves could propagate in empty space.
- The theoretical prediction was developed by James Clerk Maxwell, who unified electromagnetism in the famous four equations (flux and circuitation of the electric and magnetic field) that bear his name.

EM Fields changing in time

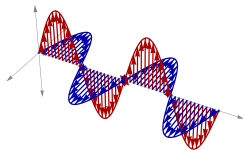
- The discovery of Faraday can be summarized as follows
- **An electric field is induced in any region of the space where there is a magnetic field that varies over time**
- Maxwell also provided a second result, perfectly symmetrical to that of Faraday:
- **A magnetic field is induced in any region of space in which there is an electric field that varies over time.**

propagation direction

- The propagation speed of electromagnetic waves depends on dielectric constant and magnetic permeability.

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \cong 3 \times 10^8 \text{ m/s.}$$

- Electromagnetic waves are *transverse* waves: \vec{B} and \vec{E} are always perpendicular to the propagation direction, and to each other
- \vec{E} , \vec{B} and the propagation direction \vec{c} constitute a right-handed triplet.
- The intensity of \vec{E} and \vec{B} are proportional to each other.



Photon energy - speed of light

The energy of a **photon** is given by the Planck formula

$$E = h \cdot \nu$$

where $h = 6.626 \times 10^{-34} J \cdot s$ is Planck's constant and ν is the frequency of radiation.

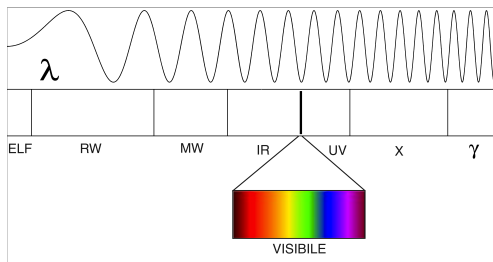
The speed of light is a function of frequency and wavelength:

$$c = \lambda \cdot \nu$$

EM spectrum

Electromagnetic wave spectrum

- Visible light is a particular type of electromagnetic wave, other being: radio waves, microwaves, ultraviolet and X-rays.
- The **spectrum** of electromagnetic waves is the set of electromagnetic waves ordered by **frequency**.



Extremely Low Frequency ELF and radio waves

- waves are located in the lower part of the spectrum, $\nu < 3\text{KHz}$ ($\lambda > 100\text{km}$). Household appliances and power lines work in this frequency range (50Hz in Italy, 60Hz in USA).
- Radio waves have wavelengths between 100km and 1m , corresponding to frequencies from 3KHz up to 3THz (10^{12}Hz). For radio broadcasts different wavelength bands (intervals) are used depending on the application: we speak of "short waves", "medium waves", and "long waves".
- Radiofrequency ablaters in medicine: an electrode is inserted into the tumor, and the electric current generated inside it heats the tumor until the cells die as a result of a thermal effect.

Microwaves MW

- wavelengths between tens of centimeters and a millimeter.
- The microwaves are used for telephone communications, for mobile telephony ($\nu = 900, 1800 MHz$) or for Wi-Fi equipment ($2.45, 5 GHz$).
- Radars emit MW pulses reflected by planes, ships and other large objects.
- The interval of time between impulse emission and reception allows to determine the distance of the obstacle and to locate its position.
- In microwave ovens, electromagnetic radiation acts on water molecules and on other polar molecules contained in food.
- The oscillating electric field exerts a force on these molecules, which increase their average level of agitation \Rightarrow kinetic energy \Rightarrow temperature.

Infrared IR

- wave lengths from 1mm to $7 \times 10^{-7}\text{m}$ are produced by sources at $T \simeq 300^\circ\text{K}$: all warm (living) objects emit in the infrared.
- Infrared has a wide applicability in Medicine, mainly to provide localized heat in organs and tissues (e.g. muscles and tendons).
- Pulse oximetry: a probe generates light beams in the red and infrared field, and a photocell that receives light after the beams have passed through the skin.
- Oxygenated hemoglobin absorbs light in red-infrared wavelengths: from the ratio of emitted to absorbed light, the instrument is able to calculate the amount of oxygen in patient's blood.

Visible light

- a typical incandescent lamp emits in the infrared, visible and ultraviolet.
- The visible radiation consists of electromagnetic waves that we perceive as light.
- This part of the spectrum is between the wavelength of $7 \times 10^{-7}m$ (red), and $4 \times 10^{-7}m$ (violet).
- At each wavelength in this interval our brain associates a color of the rainbow; outside this range, our eye is blind.
- During evolution, organs able to perceive visible light have developed: this because most of the light emitted by the sun, and reflected by the various objects on earth, is found in this frequency range.
- Visible light also finds applications in medicine. Treatment with blue light (photodynamic therapy) is applied in premature babies who suffer from jaundice (excess of bilirubin).

Ultraviolet UV

- between $4 \times 10^{-7}m$ and $10^{-8}m$ (10nm).
- Ultraviolet photon energy is sufficient to interact with molecular bonds and promote different chemical reactions (**ionizing radiation**).
- Exposure to Sun UV rays stimulates the production of melanin, causing tanning.
- Excessive exposure to ultraviolet rays can be dangerous for the skin and eyes, causing serious damage, and promoting the onset of skin tumors (melanomas).
- In astronomy, the detection of ultraviolet rays allows to study some characteristics of the Sun and the galaxies
- In the medical field, ultraviolet rays are used for the treatment of psoriasis and vitiligo.
- Furthermore, they are used to sterilize sanitary material, since they are able to destroy germs if they are subjected to irradiations of sufficient intensity.

X-rays and gamma (γ) rays

- they are the most energetic electromagnetic radiations, and together with the ultraviolet rays they constitute the **ionizing radiations**.
- X-rays have wavelengths between $10^{-8}m$ and $10^{-12}m$: they can be produced spontaneously in *radioactive decay* processes, or are artificially generated by specific devices (the X-ray tubes) with numerous applications in the biomedical field, both diagnostic and therapeutic.
- At wavelengths $< 10^{-12}m$ we have gamma rays.
- They are spontaneously emitted from nuclei during radioactive transformations and nuclear reactions, in particular fusion reactions taking place inside the stars, that reach the Earth as *cosmic radiation*.
- Gamma rays are also produced when high-energy electrons hit a target in a particle accelerator.

exercise

electrons in a magnetic field

- an experiment is designed to measure the magnitude of a uniform magnetic field using electrons
- they are accelerated from rest through a potential difference of 350 V.
- The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm.
- If the magnetic field is perpendicular to the beam,
 - 1 what is the magnitude of the field?
 - 2 What is the angular speed of the electrons?
 - 3 What is the period of revolution of the electrons?

electrons in a magnetic field

- by the end of the acceleration phase, the electron will have 350 eV of energy, knowing the charge $e = -1.6 \cdot 10^{-19} C$ that can be converted to $5.6 \cdot 10^{-17} J$
- given the mass of the electron of $9.11 \cdot 10^{-31} Kg$, we can obtain its speed as $\frac{1}{2}mv^2 = E_k \Rightarrow v = \sqrt{\frac{2E_k}{m}}$. this is equal to $1.11 \cdot 10^7 m/s$
- remembering the Lorentz force relationships
 $R = \frac{mv}{qB} \Rightarrow B = \frac{mv}{qR}$. replacing with the numerical values we obtain: $-8.417 \cdot 10^{-4} T$
- the angular speed (or pulsation) can be obtained as
 $\omega = \frac{q}{m}B = \frac{v}{R}$. This has value $1.478 \cdot 10^8 rad/s$
- the relationship between angular speed and period in a rotational motion is $T = \frac{2\pi}{\omega}$. this has value $4.25 \cdot 10^{-8} s$