

Optics

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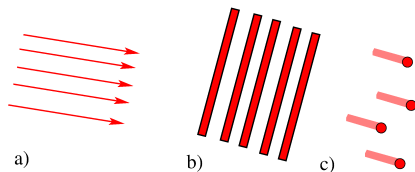
DIMES

Introduction

Optics

- Optics is one of the oldest fields of Physics, following closely astronomy and geometry
- it concerns itself with the properties of visible light and is usually divided in two subfields:
 - **Geometrical Optics**, that studies the properties of simple optical instruments and those composed out of them (mirrors, lenses, microscopes, telescopes, etc...)
 - **Physical Optics**, that studies the properties connected to the nature of light and its interaction with matters.

Optics



- a) Approximation as light rays (geometrical optics)
- b) propagation of light as waves fronts (interference and diffraction)
- c) particle representation of light; photon matter interaction (non linear optics)

Geometrical Optics

- Geometrical Optics approximates the propagation of lights as composed of simple straight lines.
- this is based on the idea of light rays, and it was known already in ancient Greece
- they believed that the light rays were emitted by the eye, and vision occurred when those hit an object
- described reflection, refraction, and how to use them to construct the most important optical instruments

Physical Optics

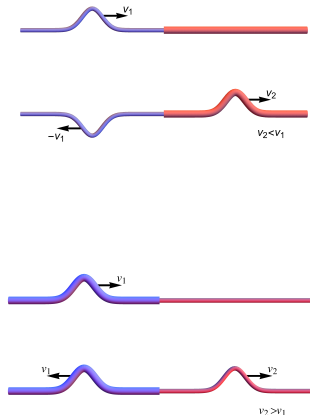
- Physical Optics studies (for our purposes) a) the phenomenons of interference and diffraction, due to how the electromagnetic wave interact with objects of size similar to the wavelength. b) laser emission
- Physical Optics is often subdivided in
 - **Waves Optics**, that studies the consequences of the wave nature of light
 - **Photon Optics**, that studies the consequences of the particle nature of light (with energy $E = h\nu$)

Geometric Optics

Basic assumptions

- The basic assumption of Geometric Optics is: **Light always travel in straight rays**
- this allows us to simplify a lot of very difficult calculations in very simple ones
- no all rays are going to be parallel to each other: from a point source the light rays are going to be divergent
- in most cases we are going to assume a plane wave, where all the rays travel parallel
- in most cases, when we describe far away source, the plane wave is a reasonable association
- this is the case for example for light rays coming from the sun

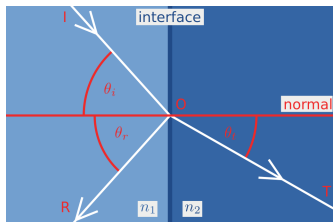
Waves reflection and transmission



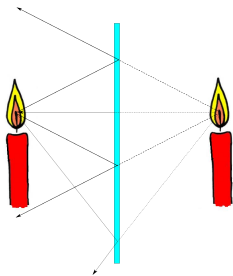
- when a wave impact a threshold of mediums with different speed, there is reflection and transmission
- the amount of reflection and transmission depends on the
- $\phi(x, t) = A \sin(\omega t + kx)$
- $A_{\text{reflected}} = A \cdot r = A \cdot \frac{k_1 - k_2}{k_1 + k_2}$
- $A_{\text{transmitted}} = A \cdot t = A \cdot \frac{2k_1}{k_1 + k_2}$
- $r^2 + t^2 = 1$

Reflection and Transmission

- when a ray of light impact an interface between two mediums, it get partially reflected and transmitted
- if there is not dispersion, the energy of the reflected wave and the transmitted one should be equal to the energy of the incident one
- all the 3 rays lays on a single plane
- the angle of reflection is always equal to the angle of incidence
- reflection can also be imagined as an elastic collision of the photon against the surface

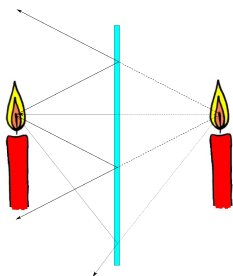


Mirrors



- mirrors reflect light shining on them independently of the frequency
- the angle of reflection is identical to the angle of incidence
- the rays of light reflected by the mirror diverge from it as if they were coming from a single point “behind” the mirror
- our eyes (and brain) reconstruct this as if there actually was an object on the other side of the mirror

Mirrors



- this is called **virtual image**
- when the rays are actually coming from a single point (be either for the presence of an object or the effects of a lens) we talk about **real image**.
- the perceived distance of the **virtual image** from the mirror is the same as the distance between the real object and the mirror
- if the mirror is flat, the perceived dimensions of the virtual object are the same of the real one (accounting for perspective)

Index of refraction

- once a ray of light enters inside a medium with a different speed of light, there is a change of the angle of incidence
- this effect is called **refraction**
- it was quantified in 1621 by Willbrord Snell
- the relationship between the incident angle and the refracted one depend on the relative speed of the wave (light) in the two mediums.
- in the case of light this is usually referred to using the index of refraction n

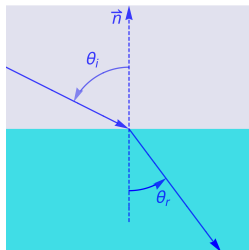
$$n = \frac{c}{v} = \sqrt{\epsilon_r \mu_r}$$

- where c is the speed of light in vacuum ($299'792'458 \text{ m/s}$) and v the speed of light in the medium

Index of refraction

Material	index of refraction
vacuum	1
air	1.000293
water	1.333
Window glass	1.52
Diamond	2.42
Moissanite	2.65
aqueous humor	1.34
lens	1.386 - 1.406
Cornea	1.376

Snell Law



- the incident and refracted ray of light lay in the same plane
- the incident and refracted ray of light lay on the opposite side of the normal to the surface separating the two mediums
- we can call θ_1 and θ_2 the angles that these rays of light have with the normal
- we can call n_1 and n_2 the index of refraction of the two mediums
- Snell's law connect the angles and the refraction indexes as such:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

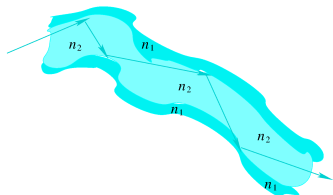
critical angle of total reflection

- if the light goes from a high refraction index medium to a low one (such as from glass to air), the resulting angle of the refracted light is going to be greater
- if the incidence angle is big enough, the refracted angle would be greater than 90° .
- this means that the light is completely reflected and no transmission happens.
- we talk about **complete internal reflection**
- the incident angle at which the refracted angle would be 90° is called critical angle θ_C

$$\sin \theta_C = \frac{n_2}{n_1} \cdot \sin 90$$

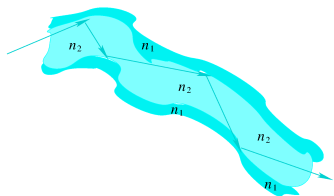
- this can be a more complete reflection than the one performed by a traditional mirror

Optic Fibers



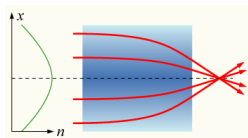
- Optic fibers are thin wires of made of materials with high refraction index
- this means that light that travel through them incurs in total internal reflection
- the wire can bend (while still smooth) and direct the light in any direction is needed, with minimal dispersion
- today most high-speed communication channels uses this technology, thank to its speed and efficiency

Optic Fibers



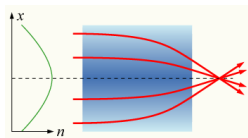
- in the medical field they are used in endoscope, colonoscope and bronchoscope.
- they allow to obtain a direct image from the inside of the patient with reduced discomfort and danger
- being completely symmetrical they allow to both send light to illuminate and return it back for observation

Refractive index gradients (GRIN lenses)



- When light comes at a very low angle on materials with non homogeneous refractive index, it get deviated several times
- this appears in the form of bending of the ray of light toward the highest refractive index area
- this can be used to construct lenses based on refractive index gradients rather than on geometrical shape
- this is the origin of most mirages and *fata morgana* (due to changes in refractive index with temperature and density).

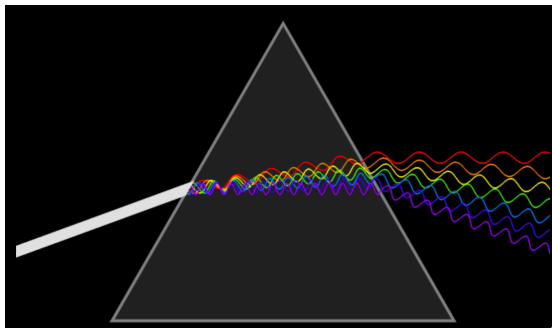
Refractive index gradients (GRIN lenses)



- it is also one of the way that the eye's lens works: it has a gradient of refractive index, from 1.386 at the edges to 1.406 in the center.
- this, alongside the change of shape during accommodation, helps the convergence of light inside the eye.
- most of the optical power is still due to the air-cornea interface, accounting for 80% of the optical power.
- during ageing the proteins in the lens stratifies and creates artifacts
- this process also stiffens the lens reducing the ability to adjust its focus

dependency from λ

- Almost all properties of the medium (in this case the refraction index n) are usually dependent on the wavelength
- this is how prisms works
- this can lead to chromatic aberrations in **optical instruments**.
- this is also how **fluorescent microscopes** works



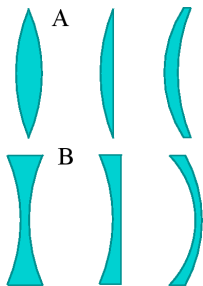
simple optical instruments

- an optical instrument is one that modifies the trajectory of light rays
- the principal component of optical systems are:
 - **mirrors**, that deflect light in a different direction
 - **lenses**, curved surfaces of transparent material (usually glass) that deflect light
 - **prisms**, crystal with different properties for various wavelength, usually used to separate the various components

Thin lens

Thin lenses

- thin lenses are disks of a transparent material (usually glass), where one (or both) face is not flat but has a spherical convexity or concavity
- combining flat side, convex and concave ones, one obtains six major kinds of lens
- lenses are divided in **convergent** and **divergent** lenses, based on the effect on a series of parallel rays of light
- the **convergent** lenses are thicker in the center and thinner at the edges
- **divergent** lenses are thinner in the center and thicker at the edges
- is the lens is small enough, all the lenses of the same family behave in the same way, so to study them we can consider the typical ones



- Convergent lenses (a) and divergent ones (b).
- the convergent lenses are: biconvex, plano-convex, positive meniscus
- the divergent lenses are: biconcave, plano-concave, negative meniscus
- the axis of the lens or optical axis connects the two centers C_1 and C_2 of the spheres that compose the lens

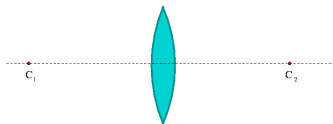


image formation

- to determine the image (real or virtual) formed by a lens, one can determine the new position of any point of the original image in the new one
- to do this one uses some geometrically convenient rays, starting from the point of interest
- using any two of these, the position of the destination point can be determined
- there are three main rays used for these geometrical construction, any two will do

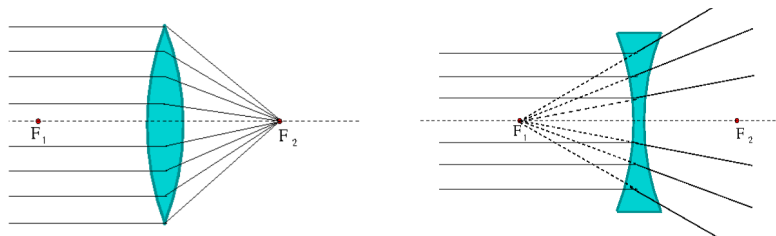
image formation

- **one ray that goes through the center of the lens.** due to symmetry reasons, this ray is not changed and goes straight without modifications
- **one ray that parallel to the optical axis.** This ray will be deviated toward the focal point (so called because in convergent lenses this point can be used to start fires)
- **one ray that goes through the focal point.** this ray will come out parallel to the focal axis (in symmetry with the previous case)

focal point

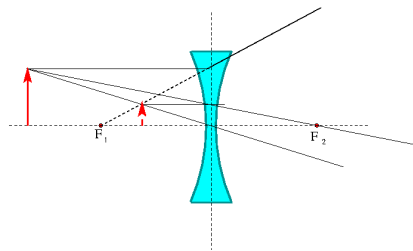
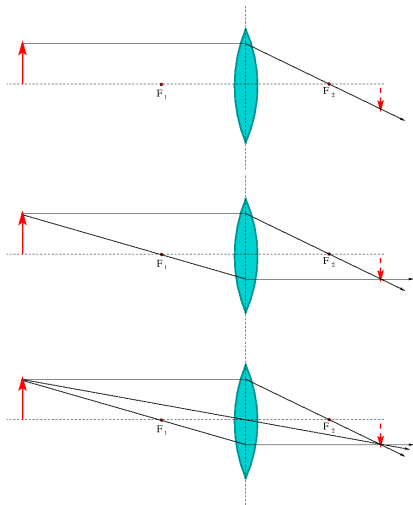
- for this construction the focal point is fundamental.
- there are two focal points, one for each side of the lens. the more the surface is curved, the shorter the distance between the focal point and the lens
- if the lens is symmetrical, the focal points will be at the same distance from the lens
- The reciprocal of the focal length, $1/f$, is the optical power of the lens.
- If the focal length is in metres, this gives the optical power in dioptres (inverse metres).

focal point



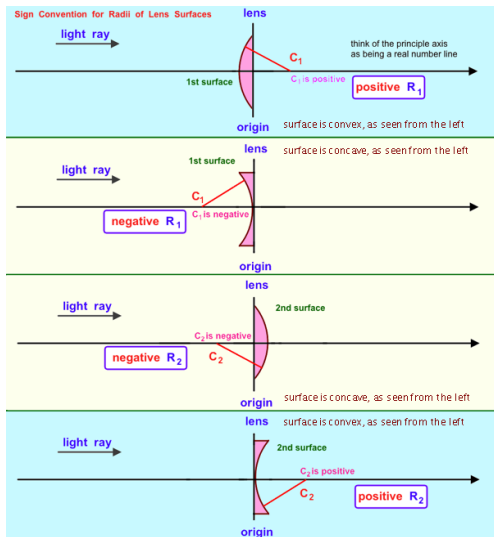
- The focal point of a convergent and a divergent lens
- the image in the convergent focal point is real (the rays actually meet there)
- the image in the divergent focal point is virtual (it appears as if they comes from a single point, but they don't actually converge there)

Geometrical construction of the image



Geometrical construction for
convergent and divergent lenses

lens curvature



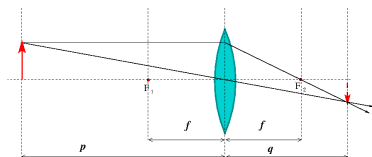
Lensmaker's equation

- the relationship between the geometry and the refractive index of the lens and the position of the focal point is described by the lensmaker's equation.
- in the assumption of thin lenses, it can be simplified as:

$$f^{-1} = (n - 1)(R_1^{-1} - R_2^{-1})$$

- f is the focal length of the lens,
- n is the refractive index of the lens material,
- R_1 is the radius of curvature of the lens surface closer to the light source,
- R_2 is the radius of curvature of the lens surface farther from the light source

thin lens formula



- given an object at distance p from the lens, put in front of a lens with focal distance f , the distance at the image will form is given by the **thin lens formula**

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

- if one replace $p - f$ with x and $q - f$ with y , this gives the “newtonian formulation” as

$$xy = f^2$$

Magnifying lenses

Magnifying lenses

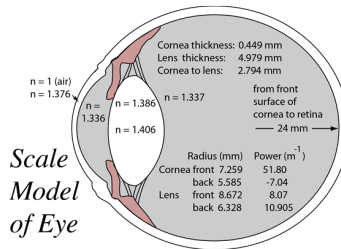
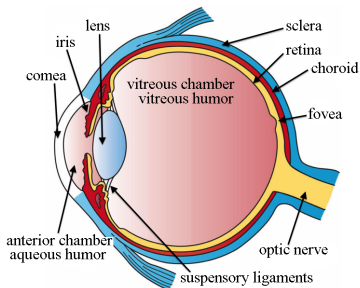
- the magnifying lens generate a bigger image of the original object
- they are defined as **simple microscopes**
- the lens inside the eye is one of such lenses
- given a focal point for a lens, the relative size of the generated image is given by the magnification factor G :

$$G = \frac{q}{p}$$

Magnifying lenses $G = q/p$

- we can get a greater magnification by getting the object closer to the focal point
- or we could change the shape of the lens to get the focal point closer to the object
- this is how our eye works
- closer than 25 cm the lens is not able to focus properly anymore
- with the magnifying lens we can focus even closer, achieving greater magnification

Structure of the eye



Magnifying lenses

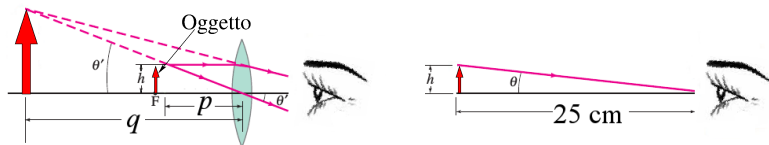


Image formed by a magnifying lens compared to the one formed in the human eye

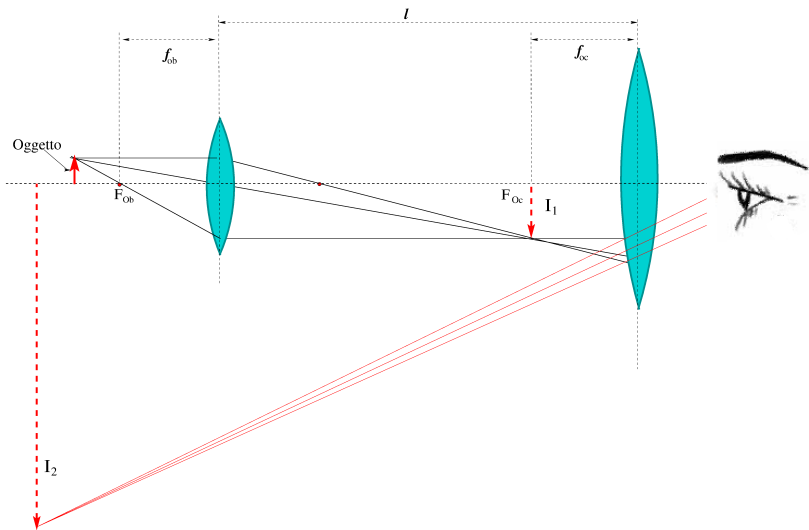
Composite microscope

- a composite microscope is composed of two lenses, one after the other, both convergent.
- one is the ocular and one is the objective lens
- the object to be observed is positioned in the focal point of the objective, and generate a strongly magnified image in the focal point of the ocular
- in real microscopes there are other intermediate lenses that are used to correct **aberrations**
- the magnification power P_I of a real microscope can reach up to 1'000x, and depends on the geometrical characteristics of the microscope

$$P_I = \frac{25 \cdot l}{f_{ob} \cdot f_{oc}}$$

- l distance between the lenses, f_{ob} focal distance of the objective and f_{oc} focal distance of the ocular

Microscope (composite)



physical resolution limits - aberrations

- from a mathematical perspective we could achieve infinite magnification, but in reality this is limited by two factors, aberrations and diffraction
- aberrations are distortions of the image due to the non ideal properties the lenses
- monochromatic aberrations are due to geometrical (shape) limitations, such as non zero thickness
- chromatic aberrations are due to difference in the refractive index for different wavelengths

physical resolution limits - diffraction

- the light is not really a geometrical set of perfect rays, but rather a series of wavefronts
- this creates defects whenever it goes through an opening.
- these defects limits the maximum resolution one can achieve

$$R = \frac{n \sin \alpha}{0.61 \cdot \lambda}$$

- R is the inverse of the minimum distance (in cm) between two objects that can be distinguished
- λ is the light wavelength
- α is the opening angle of the light that can enter the objective lens
- with visible light I have a maximum resolution of $0.2 \mu m$, (cells and cell sub-components, even bacteria, non viruses).
- this will be explained better in the next section

Physical Optics

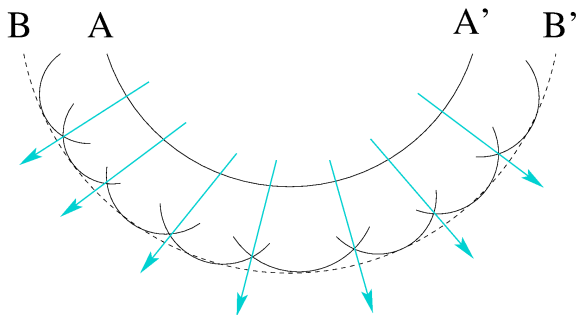
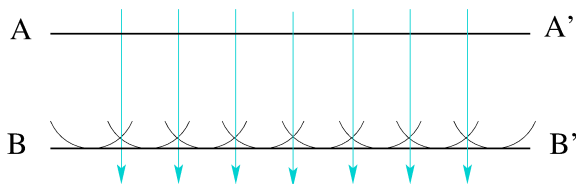
Limitations of Geometrical Optics

- as we discussed, geometrical optics is based on the idea that light moves as straight lines
- we know that light is a wave, and that approximation is only valid for far away light sources, and small lenses
- in general one has to consider the complete evolution of the wavefront. to do this one has to employ the physical optics
- this means that there is no single, simple direction, but it can vary from position to position and over time

Huygens principle

- In the XVII century the study of light waves brought physicist Christian Huygens to define a principle to explain the movement of wavefront in any medium
- **Any point of the medium hit by a wave can be imagined as emitting a spherical wave with the same frequency**
- the movement of the wave front can be calculated by combining all these generated waves
- these combination is between waves of similar frequency but different phase, so they do interference with each other.
- the resulting propagation of the wavefront is the one observed also in the geometrical optics
- it is also one of the main methods to simulate the propagation of waves in complex media geometry and with non trivial effects

Huygens principle



Interference and diffraction



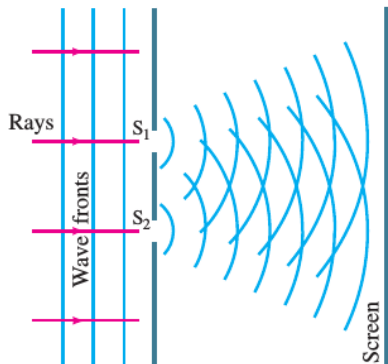
Interference and diffraction

- In general one cannot distinguish between interference and diffraction
- the term diffraction is used to describe the behavior of light nearby opening or obstacles
- this effect is generated by the interference of the waves at the opening

Interference and diffraction

- if the sources are incoherent then interference is not possible
- when two waves with the same wavelength interact they can have constructive or destructive interference (or partial, intermediate effects)
- this wave-like behavior is the basis for the Young's experiment

Young Experiment



λ light wavelength

d distance between the slits

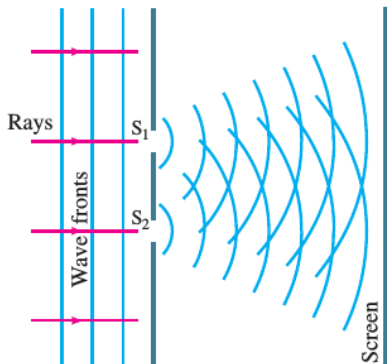
x distance between observed maxima

L distance between the slits and the screen ($L \gg d$)

$$\frac{\lambda L}{d} = x$$

it is the basis for the spectroscopy experiments

Young Experiment



- rule for maxima:

$$d \cdot \sin(\theta) = m \cdot \lambda$$

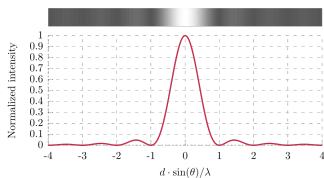
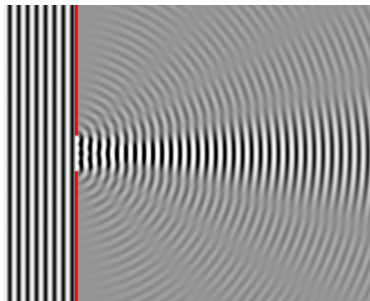
$$m = 0, \pm 1, \pm 2 \dots$$

- rule for minima:

$$d \cdot \sin(\theta) = (m - 1/2) \cdot \lambda$$

$$m = 0, \pm 1, \pm 2 \dots$$

Diffraction



Airy disks

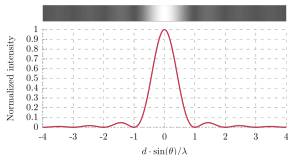
$$d \sin \theta_{\min} = n\lambda$$

d width of the slits

n number of slits

λ wavelength of the light

θ_{\min} first minima angular width



Resolution limits

$$r = \frac{1.22\lambda}{2n \sin \theta}$$

r minimum length that can be resolved

n medium index of refraction

λ wavelength of the light

θ angular opening of the lens (relative to the direction of observation)

- Resolution is inversely proportional to the angular opening.
- Angular opening is proportional to the amount of light that can be collected

Laser

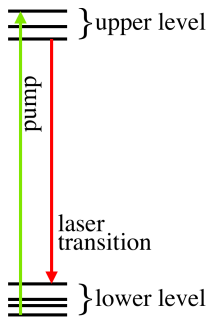
Laser

- LASER stands for “Light Amplification by Stimulated Emission of Radiation”
- was proposed by Einstein in 1917
- was developed in the 1960 for the first time
- it is a different mechanism of light emission compared to normal light sources
- all the light generated is coherent, both spatially and temporally
- is based on the phenomenon of stimulated emission, a quantum effect

Complex refractive index

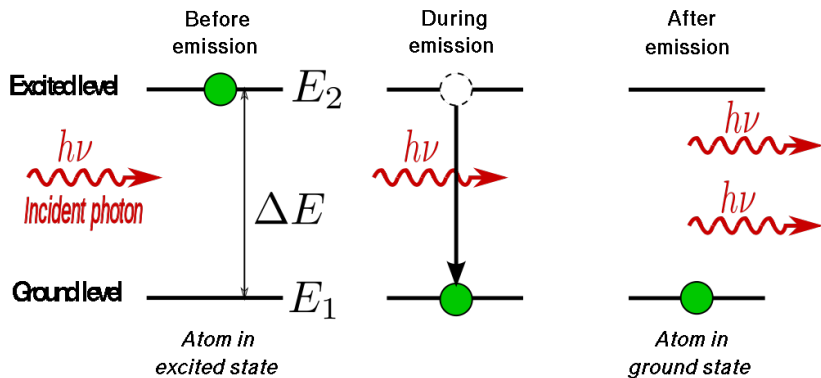
- a material that absorb light can be modeled using a complex refractive index $n = n_r \pm ik$
- the terms k is usually negative and represent light absorption in the medium
- this absorption is exponential, with a typical free path $\alpha = \frac{4\pi k}{\lambda}$, where λ is the wavelength of light
- some materials, under certain conditions, have a positive k , and are called gain mediums
- this means that they exponentially increase the intensity of light passing through them
- this requires energy provided to the medium from an external source
- this is the basis of the phenomenon calle stimulated emission

Stimulated Emission



- an energy source is used to excite the electron in a material
- this material needs to have specific properties
- they can be crystal, semiconductors, gases, etc...
- once the excited medium is hit by a photon, it emits a second one with the same wavelength and phase as the first one
- this process converts external energy in light amplification

Stimulated Emission



$$E_2 - E_1 = \Delta E = h\nu$$

Laser applications

- Laser technology underpins a huge amount of modern technology
- fiber optic communication (with the ability of short pulsations)
- high precision cutting
- additive manufacturing using high precision melting
- spectroscopy and interferometry
- bar code readers
- optical disk readers

Laser medical applications

- microscopy (fluorescence and confocal)
- laser surgery
- laser ablation (Laser-induced interstitial thermotherapy for tumor treatment)
- ophthalmoscopy